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# THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE

Mount Wilson Charresters of the Carnegie

EDWIN B. FROST Yerkes Observatory of the University of Chicago

HENRY G. GALE

Ryerson Physical Laboratory of the

University of Chicago

### NOVEMBER 1930

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NUMBER 4

### U CEPHEI: AN ANOMALOUS SPECTROGRAPHIC RESULT

#### By EDWIN F. CARPENTER

#### ABSTRACT

Thirty-four spectrograms of the bright component of the eclipsing variable U Cephei, in spite of broad, shallow lines and the resulting large probable error of a single observation (13 km/sec.), yielded, by reason of a large range in velocity (220 km/sec.), a satisfactorily defined velocity-curve, the elements from which (Table VI) indicate an orbit of abnormally high eccentricity (0.474) whose major axis is inclined approximately 65° to the line of sight. Earlier photometric data, largely influenced by a shallow secondary minimum near mid-phase, have indicated a circular orbit, but constancy of duration of eclipse and the nature of the variation of the period seem to preclude a reconciliation of the two sets of data by the assumption of a rotation of the line of apsides which should bring it into coincidence with the line of sight at the photometric epoch. The period is shown to increase steadily, with an oscillation of a few seconds' amplitude and an irregular period of about a dozen years. The usual methods for photometric orbits are extended to include cases where spectrograms have indicated orbits of high eccentricity.

Since its discovery by W. Ceraski in 1880, U Cephei has been one of the more thoroughly observed, photometrically, of the eclipsing variables, a long series of measures by various observers culminating in the extensive work of R. S. Dugan,<sup>I</sup> which yielded an orbit of high internal consistency. It therefore seems especially desirable to supplement these data with a spectroscopic orbit, and I was able to take spectrograms for this purpose at the Lick Observatory in 1923–1925, for which opportunity I am indebted to the kindness of Dr. Aitken. The only spectroscopic data available to me regarding this star, besides the H.D. classification<sup>2</sup> of primary and secondary

<sup>&</sup>lt;sup>1</sup> Princeton Contributions, No. 5. Reference is made to this memoir for a very complete summary of earlier observations.

<sup>&</sup>lt;sup>2</sup> See also A. J. Cannon, Popular Astronomy, 25, 314, 1917.

components (Ao, Ko) and an earlier description of the spectrum at various phases by S. Blajko, is V. M. Slipher's report of a range of 90 km/sec. between two plates taken in 1907. Some other observers, however, have photographed the spectrum, and it is perhaps due to the discouraging character of the lines that no spectrographic orbit has heretofore been computed; but a large range in velocity fortunately renders the poor definition of the lines rather innocuous.

#### THE SPECTROGRAPHIC DATA

In view of the unexpected orbital eccentricity disclosed by the spectrograms, it appears well to describe the observations and reductions in rather more detail than would be required ordinarily. The plates were made with the 36-inch refractor, with use of a singleprism spectrograph with a 16-inch camera combination, giving a dispersion of 12.8 mm between  $H\beta$  and  $H\delta$ . Exposures were generally 30-60 minutes in length, though a few were shorter. Only wide and diffuse hydrogen lines appear, three of which were ordinarily usable for the determination of velocity. Owing to the faintness of primary minimum, observations were confined to the phases of maximum light (6.8 mag.), and showed no trace of secondary spectrum, just as would be expected from the considerable range in brightness (2.3 mag.). From a plate taken with an exposure of 150 minutes during primary minimum, A. H. Joy very kindly provided a velocity of the faint component, based on seven lines. In conformity with the photometric data, a nearly circular orbit was expected, and it was intended to use only eighteen or twenty spectrograms, but when a preliminary plot showed evidence of considerable eccentricity, several more plates were taken at critical phases, making a total of thirty-four usable plates, including the one from Mount Wilson.

All of the spectrograms except Joy's were measured on a Gaertner comparator by Mrs. Carpenter (to whom I am also indebted for the greater part of the computing as far as orbit III), and many of the plates were measured by myself as well, the measurer being in no case aware of the phase of the plate. Table I presents the observational data. Since  $H\beta$  and  $H\gamma$  were always measured and  $H\epsilon$  could

Annales de l'Observatoire de Moscou (2d ser.), 5, No. 9.

<sup>&</sup>lt;sup>2</sup> Astrophysical Journal, 25, 284, 1907.

be used only on exposures strong enough to show  $H\delta$  as well, column 7 indicates the lines used for the velocities for each plate except Joy's, whose line identifications were not revealed. The weights

 $\begin{tabular}{ll} TABLE & I \\ Spectrographic Observations of U Cephei \\ \end{tabular}$ 

Plate No.	Nor- mal No.	Date		G.M.T.	J.D. 2420000+	Phase from Obs. Prim. Mini- mum*	No. Lines	Wt.	Veloc. Km/Sec.	0-C
12794	8	1923 Jul.	18	23 <sup>h</sup> 35 <sup>m</sup>	3619.983	2 do34	4	2	+126	- 6
Y 11942 t	12		21	22 10	3622.924	2.480	7	3	- 4	+12
12807	7		23	21 6	3624.880	1.949	3	1	+134	+38
12808	7		23	22 2	3624.918	1.989	3	1	+106	- 8
12809	8		23	22 47	3624.949	2.019	3	1	+126	0
12810	8		23	23 14	3624.968	2.039	3	2	+126	- 9
12843	3	Aug.	6	21 39	3638.902	1.012	3	2	- 74	- 20
12844	3		6	22 38	3638.943	1.052	3	2	- 63	-11
12845	3		6	23 51	3638.994	I.102	3	2	- 53	- 4
13102	1	1924 Jan.	7	17 34	3792.731	0.282	3	1	- 65	- 3
13446‡	5	Jun.	I	17 46	3938.740	1.703	3	2	+ 18	- 7
13447‡	5		I	18 29	3938.770	1.733	3	1	+ 58	+26
13542	4	Jul.	28	17 20	3995.720	1.346	3	2	- 27	+ 2
13543	4		28	18 59	3995.791	1.416	4	2	- 31	-10
13549	3		30	19 13	3997.801	0.935	3	2	- 63	- 5
13566	9	Aug.	5	22 57	4003.956	2.102	3	2	+152	+ 4
13584	10		21	0 53	4019.036	2.229	3	2	+138	+11
13715	3	Oct.	30	22 32	4089.939	0.835	3	2	- 54	+ 8
13716	3		30	23 47	4089.989	.885	3	2	- 50	+10
13756	2	Nov.	19	16 19	4109.680	.633	2	1	- 76	- 8
13757	2		19	17 56	4109.747	0.700	2	1	- 64	+ 2
13832	IO	1925 Jan.	2	15 43	4153.655	2.224	2	1	+108	-18
13833	II		2	17 20	4153.722	2.293	4	2	+ 57	-21
13846	1		5	15 58	4156.665	0.247	3	2	- 46	+15
13847	1		5	18 25	4156.767	0.356	3	2	- 85	-19
13899	6	Feb.	18	16 5	4200.670	1.877	3	1	+ 93	+31
13901	7		18	18 14	4200.760	1.967	3	1	+ 91	-12
13939	I	Mar.	13	22 55	4223.955	0.229	3	1	- 79	-21
14007	4	May	23	21 32	4294.897	1.375	2	$\frac{1}{2}$	+ 19	+45
14008V	4		23	22 52	4294.952	1.430	2	1 2 1 2 1 2	- 26	- 6
14008W	4		24	0 20	4295.014	1.492	2	1/2	+ 22	+34
14022	5	Jun.	5	17 55	4307.746	1.770	4	3	+ 37	- 4
14023	6		5	20 56	4307.872	1.896	4	3	+ 54	-24
14024	7		5	22 30	4307.937	1.961	4	3	+106	+ 4

<sup>\*</sup> Based upon earlier observations of Campbell (H.B., No. 762).

listed in column 8 were assigned at the time of measurement, and are based upon the appearance of the plate and the consistency of its

<sup>†</sup> Mount Wilson observation of secondary component.

<sup>‡</sup> Spectrograph provided with dense prism, dispersion = 18.1 mm between  $H\beta$  and  $H\gamma$ .

results. Table II, which requires no explanation, shows the combination of the observations into normal places.

The period of U Cephei, which will be discussed in more detail on a later page, has long been known to be variable. In reducing the spectrographic observations the latest available light-elements given

TABLE II Spectrographic Normal Places

No.	Wt.	Phase from Obs. Prim. Minimum	Velocity Km/Sec.	O-C
1	0.50	od283	- 67.8	- 5.3
2	0.17	.667	- 70.1	- 2.7
3	1.00	0.971	- 59.5	- 2.7
4	0.42	1.395	- 19.7	+ 4.2
5	.50	1.741	+ 34.2	+ 1.1
6	-33	1.892	+ 63.5	-13.0
7	.50	1.968	+107.8	+ 3.5
8	.42	2.034	+125.7	- 4.0
9	.17	2.100	+152.0	+ 3.0
0	.25	2.225	+128.0	+ 6.8
1	.17	2.291	+ 57.0	-20.9
12	0.25	2.479	- 4.I	+12.1

by Leon Campbell<sup>1</sup> as best satisfying observations from 1905 to 1922 were used:

but his later observations<sup>2</sup> showed that during the interval of the spectrographic observations it would be more satisfactory to use

J.D. 
$$2424804.604 + 2^{d}4929507E$$
.

The effect of this change of the period upon the reduction of the spectrograms is inconsequential, although there is a slight progressive change in phase, which is shown in Table III. While the phases used originally are kept in Tables I and II, in the further discussion the effect of epoch has been practically removed by the application of a mean correction of -0.02 days to the phases from observed primary minimum.

A plot of the individual observations, reduced by the method of Lehmann-Filhés, yielded preliminary elements which, with the exception of the period, were improved by a least-squares solution,

Harvard Bulletin, No. 762, 1922.

<sup>2</sup> Ibid., No. 842, 1927.

with employment of Schlesinger's usual formulae.<sup>1</sup> The corrections are given in the second column of Table V. Since some of the correc-

TABLE III

TRUE PHASES OF SPECTROGRAMS FROM PRIMARY
MINIMUM minus PHASES USED IN COMPUTING
THE SPECTROGRAPHIC ORBIT

J.D.			_		_						Ī		-	7	Δ Phase
2423560.			,												-0,010
2423810.															.015
2423060.		*		*											.020
2423310.															-0.025

#### TABLE IV

#### PRELIMINARY ELEMENTS

$P = 2^{d}492901$ (adopted for the	e = 0.451
solution)	$\omega = 20^{\circ}3$
$\gamma = -5.0$ km/sec.	T = J.D. 2423966.682
K = 115.0  km/sec.	

tions seemed rather large, a second adjustment was carried out, the results of which are shown in columns 3 and 4. A further attempt

TABLE V

Corrections to Spectrographic Elements Given by
Least-Souares Solutions

		Solution									
	I	1	I	I	п	IIIa					
	Corr.	Corr.	P.E.	Corr.	P.E.	Corr.					
γ, km/sec K, km/sec	-10.1	-0.9 +3.2	2.6	+1.5		-1.9 +5.4					
$egin{array}{lll} \omega & & & & & & & & & & & & & & & & & & $	+ 4°5	+1°8	3°4	+0.010 -1.6 -0.000	3°3	+0.058 +3°2 -0.003					
Normal place of wt. 1, km/ sec			3.6		3.7						

was made to reduce the probable errors (cols. 5 and 6), which, as was expected, was ineffective, but the orbital elements resulting from this solution III were adopted as final.

<sup>&</sup>lt;sup>1</sup> Publications of the Allegheny Observatory, 1, 33, 1908.

The tabulated period is Campbell's for the spectrographic epoch. The eccentricity is the highest for any eclipsing binary, and is even very high for a spectroscopic binary, for which it would normally be associated with a period at least ten times as long. The velocity-curve resulting from these elements is the upper curve in Figure 1, the individual observations being represented by solid squares and the normal places by open circles the radii of which are equal to the

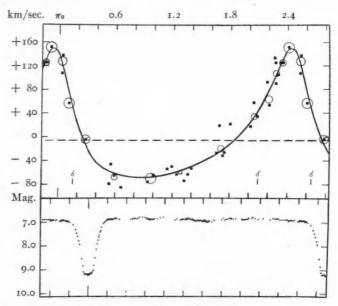


Fig. 1.—Above: Velocity-curve of U Cephei. Solid squares represent the individua observations, circles the normal places with their radii equal to the probable errors. The barred circle marks the Mount Wilson normal place. Abscissae represent phases from periastron. The computed phases of conjunction are indicated by the usual symbol. Below: The light-curve as defined by Dugan's normal places. Abscissae indicate phases from primary minimum. The two curves are adjusted to show their observed relative phases.

weighted probable errors. The Mount Wilson observation is plotted as a barred circle. Residuals from this curve for the individual observations and for the normal places are shown in the last columns of Tables I and II.

The propriety of including in these solutions Joy's velocity of the secondary star at primary minimum is perhaps open to question.

The velocity of a component at conjunction differs from the systemic velocity by  $Ke\cos \omega$ , which vanishes in general only in cases of circular orbits. In this case, however, on a reasonable assumption for the relative mass of the secondary star, this term is not greatly in excess of the uncertainty of a normal place of its weight, and in view of the lack of observations near this phase, occasioned by the faintness of the secondary, the retention of the observation seemed justified. In any event, its residual is not large, but to test more thoroughly its effect, a solution based on orbit II was carried out, by the use of the same equations of condition as for III with the omission of No. 12. This solution, IIIa of Table V, is thus directly comparable

#### TABLE VI

#### FINAL ELEMENTS

Epoch=1924.5 
$$\omega = 25^{\circ} \circ \pm 3^{\circ} 3$$
  
 $P = 2^{\circ} \cdot 4929507$   $T = J.D. 2423966.644 \pm 0^{\circ} 014$   
 $\gamma = -6.0 \pm 3.7 \text{ km/sec.}$   $a \sin i = 3,320,000 \text{ km}$   
 $K = 109.9 \pm 3.1 \text{ km/sec.}$   $a \sin^{3} i = 0.235$   
 $e = 0.474 \pm 0.022$   $m_{1}^{3} \sin^{3} i = 0.235$ 

with III. No marked change results, but the eccentricity and periastron *minus* node are both somewhat increased, a result which might have been anticipated from an inspection of the velocity-curve, this observation obviously tending to reduce slightly the asymmetry of the curve. Hence the remarks appearing below concerning orbit III apply with at least equal force to an orbit based wholly upon my plates.

#### THE PHOTOMETRIC DATA

Photometric orbits of U Cephei, representing a large faint star totally eclipsing a small bright star at primary minimum, have been derived by H. Shapley, R. S. Dugan, and R. H. Baker. Shapley used 695 photometric observations by O. C. Wendell, Dugan, 14,112 measures of his own with a polarizing photometer; and Baker, 305 extra-focal photographs made by Miss Cummings and himself.

<sup>&</sup>lt;sup>1</sup> Princeton Contributions, No. 3; Astrophysical Journal, 36, 269, 1912.

<sup>&</sup>lt;sup>2</sup> Princeton Contributions, No. 5, 1920.

<sup>3</sup> Laws Observatory Bulletin, No. 30, 1921.

<sup>4</sup> Harvard Annals, 69, 58, 1909.

The light-curves are in essential agreement, except for the greater depth of Baker's primary minimum occasioned by the late type of the secondary. Dugan's curve, as the most complete, is reproduced in Figure 1 in its observed phase with the velocity-curve above it. All three light-curves are characterized by distinct asymmetry during the primary minimum, in the sense of a more rapid rise than descent. There is some evidence for a slight retardation of the secondary minimum from the mid-phase of the light-curve. Shapley estimates an uncertain tenth-of-a-day retardation; Baker seems to regard it as fairly certain; and Dugan admits its possibility, though in the last two cases the curve is not so well covered at emergence

TABLE VII
THE PHOTOMETRIC ORBITS

	Shapley	Baker	Dugan
Epoch of observation	1902.7 Uniform	Uniform	1915.5 One-third darkened
Eccentricity (assumed)	odi:	o odi	o Slight
Duration of eclipse	.442	.414	0 <sup>d</sup> 420
Duration of total phase	0.0894	0.101	0.0797
Inclination of orbit	90°	90°	86°4
Ratio of radii, k	0.63	0.60	0.62
Semi-major axis of bright star, $r_1$	.324	.320	.322
Semi-major axis of faint star, $r_2$	0.205	0.100	0.200

from the secondary eclipse as it is elsewhere. In the opinion of these computers this lag perhaps indicates some slight orbital eccentricity, but since its effect cannot be separated from the effects of orbital orientation, on account of the shallowness of the secondary minimum, in deriving their orbits, an eccentricity of zero is assumed. Table VII presents the pertinent data of these orbits.

Apart from the question of the real shape of the orbit, there are two discrepancies between the results from the spectrographic and photometric observations: (a) the spectrographic elements predict superior conjunction  $0^d \cdot 164 = 3^h \cdot 9$  earlier than the observed primary eclipse, and (b) in consequence of the shape and orientation of the orbit, the spectrographic phase of inferior conjunction occurs  $0^d \cdot 51 = 12^h \cdot 2$  after the mid-phase occupied by the observed secondary minimum.

#### ATTEMPTS TO RECONCILE THE DATA

These inconsistencies lead to the question of the reliability of the orbits which are compared, and, in view of the independent confirmation of the photometric results, suspicion falls upon the spectrographic orbit. It is largely for this reason that this paper has been thus delayed, in the expectation of repeating the spectrographic observations at a different epoch at this observatory, but equipment not developing in this direction as it was hoped, this plan has now to be foregone. However, in justification of the spectrographic results it may be pointed out that in the instruments and reduction there was no departure from standard equipment or procedure, that the large range in velocity and the combination of results of widely separated dates into the same normal preclude any likelihood of systematic error, that the observations themselves are thoroughly consistent, showing only moderate relative dispersion, and that Joy's independent velocity behaves quite as would be expected. It is nevertheless possible that many more spectrograms could change the velocity-curve so that the four-hour discrepancy at primary eclipse would be eliminated, although it seems too much to expect that the general form of the curve could be materially altered. Removal of the discrepancy by introducing it into a least-squares solution is unsatisfactory by reason of the large resulting residuals which enter into the equations of conditions, but if the three elements primarily concerned are each changed in the optimum direction (i.e., T increased and  $\omega$  and e decreased) by four times their probable errors, the time of primary eclipse is satisfactorily predicted. The curve thus resulting from these so arbitrarily adjusted elements,

$$T = \text{J.D. } 2423966.700$$
,  
 $\omega = \text{II.8}$ ,  
 $e = 0.386$ ,

does not, however, fit the observations well, running 30–40 km/sec. too high up to a phase of one day from periastron and consistently about 20 km/sec. too low thereafter, while owing to the greater asymmetry of orientation of the orbit the secondary eclipse is worse represented than before by fully two hours. And it is doubtful if

there is any virtue in correcting the discrepancy in one eclipse if the other is not improved.

Since the photometric and spectrographic observations here discussed are separated by an interval of nine years, it is tempting to try to reconcile the two orbits on the basis of a rotation of the line of apsides, which must be consequent upon the polar flattening which is usually found in eclipsing systems of short period. The photometric observations of 1915 are then supposed to take place with the major axis of the orbit nearly coincident with the line of sight. Since the eccentricity of U Cephei is too great to be expressed manageably in series form, Russell's usual formulae for the photometric elements of an eclipsing binary with eccentric orbit<sup>1</sup> cannot be used. Instead, the variable radius vector may be introduced into the fundamental equations for circular orbits.<sup>2</sup> If the semi-major axis of the relative orbit is taken as unity, then the distance between centers of the stars is given by

$$\delta^2 = R^2 \cos^2 i + R^2 \sin^2 i \sin^2 \theta , \qquad (1)$$

where R is the radius vector and  $\theta$  has the same meaning that Russell gives it but does not vary uniformly with time. Then

$$R^2 \cos^2 i + R^2 \sin^2 i \sin^2 \theta = r_1^2 \{\varphi(k, \alpha)\}^2$$
 (2)

and, following Russell,

$$\frac{(R_1^2 - R_2^2)\cos^2 i + (R_1^2\sin^2\theta_1 - R_2^2\sin^2\theta_2)\sin^2 i}{(R_2^2 - R_3^2)\cos^2 i + (R_2^2\sin^2\theta_2 - R_3^2\sin^2\theta_3)\sin^2 i} = \psi(k, a_1).$$
(3)

Unless the principal eclipse takes place at periastron under rather special circumstances of the stellar radii, a system having the light-curve of U Cephei must have  $i=90^{\circ}$  nearly, so that as a first approximation, to be tested later, we may write

$$\psi(k, \alpha_{\rm I}) = \frac{R_{\rm I}^2 \sin^2 \theta_{\rm I} - R_{\rm 2}^2 \sin^2 \theta_{\rm 2}}{R_{\rm 2}^2 \sin^2 \theta_{\rm 2} - R_{\rm 2}^2 \sin^2 \theta_{\rm 3}} \tag{4}$$

or

$$R_{\rm r}^2 \sin^2 \theta_{\rm r} = A + B\psi(k, \alpha_{\rm r}) , \qquad (5)$$

where the R's are now included in A and B. If now the values for  $\theta$  are computed from the adopted spectrographic orbit, the usual tables for circular orbits may be used for finding the ratio of the radii of the stars, k. If equation (2) is then written for  $\theta_6$  and  $\theta_9$  and the ratio taken, a little recombination of terms yields an expression for  $\cos^2 i$ , and  $r_1$  may now be determined from (2) written for  $\theta_6$  or for  $\theta_9$ . Finally, the light-curve may be represented by (5); but, since there are now two variables, R and  $\theta$ , the former varying the more slowly, it will usually be necessary to proceed by successive approximations, especially at some distance from apastron.

Using this method, but without taking the trouble to rectify the light-curve, especially since this becomes very complex and highly

#### TABLE VIII

PHOTOMETRIC ELEMENTS FOR PRIMARY ECLIPSE AT APASTRON

Speed of orbital rotation 
$$= 13^{\circ}$$
 per year  $r_1 = 0.198^*$   $r_2 = .133^*$   $k = 0.67$   $i = 90^{\circ}$ 0

Ratio of surface brightness = 15.8

\* Semi-major axis of relative orbit = unity.

uncertain with a large eccentricity, solutions were made on the assumption of primary eclipse taking place at periastron and at apastron. The former assumption must at once be ruled out, because the resulting stellar radii are so great as to cause interpenetration at periastron, but the second assumption leads to apparently reasonable elements, as shown in Table VIII.

The ratio of surface brightness is in good agreement with the spectral types. At periastron the stellar surfaces are separated by a distance approximately equal to the radius of the larger star. Except for the asymmetry, which was not explicitly introduced, the primary light-minimum is well represented, but the shallow secondary minimum, while well represented as to depth, has a duration rather less than half the observed. Apart from this last point, which is some-

<sup>&</sup>lt;sup>1</sup> Apart from the difficulties of phase, the spectrographic elements of orbit III also result in a geometrically impossible representation of the observed primary minimum.

what involved in observational difficulties, there are four well-known tests of orbital rotation, to which the observations of U Cephei do not respond favorably: (a) The duration of eclipses should be alternately long and short as they occur alternately near apastron and periastron. (b) The occurrence of secondary eclipse with respect to primary should oscillate about the mid-phase. (c) The apparent photometric period should undergo a regular oscillation about a mean. (d) The asymmetry of light-minimum should alternately involve the descending and ascending branches in opposite sense, but since other asymmetrical characteristics might possibly effectively mask this it may be dismissed as of secondary importance.

TABLE IX

DURATION OF PRIMARY ECLIPSE PHASES BELOW VARIOUS

MAGNITUDES AT FOUR EPOCHS

	Magnitude										
	7.0	7.5	8.0	8.5	9.0						
Pickering,* 1880–1881 Pickering, 1895–1897	od33	od23	od18	od16	od12						
Wendell, 1895–1902	.47	. 26	.18	.15	.12						
Dugan, 1914-1916	0.37	0.23	0.17	0.13	0.11						

<sup>\*</sup> Data for the first three entries are taken from Müller and Hartwig, Geschichte und Literatur des Lichtwechels, 1, 28, 1918; for the last, from Dugan 3 paper to which reference has been made.

a) The duration of eclipses.—Mrs. Shapley, in her discussion of the period of U Cephei, shows that Wendell's observed times of the star passing magnitude 8.4 on the descending and ascending branches indicate very effectively that there was no change in the duration of primary eclipse at the corresponding phase in the interval from 1896 to 1911. Further confirmation of this extending from 1880 to 1916 is contained in Table IX, which gives the duration of five phases of primary minimum at four epochs. The intermediate magnitudes deserve greatest weight, since in passing through these the brightness is varying most rapidly. Clearly the duration of eclipse is substantially constant over an interval of thirty-six years, whereas an eccentricity of the order indicated by the spectrographic data implies at least twice the duration of eclipse at apastron as at periastron.

Astrophysical Journal, 44, 51, 1916.

b) The phase of secondary eclipse.—Even if the apparent photometric period appears disturbed by other causes, there should at least result from orbital rotation a periodic displacement of secondary minimum about mid-phase by an amplitude indicated in this case to be at least fifteen hours. Evidence available to me on this point is negative, the three complete light-curves discussed above showing the secondary minimum close to mid-phase, but this of itself is not very conclusive, since the interval between Wendell's epoch and Dugan's and Baker's epoch is favorable to a rotation of 180° at a rate approximating 13° per year.

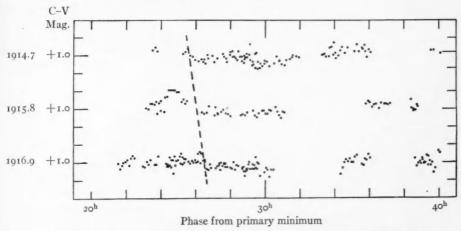


Fig. 2.—Dugan's unweighted individual observations through secondary minimum at three epochs. The ruled magnitude interval=0.2 mag.

Perturbations of this magnitude ought to show in Dugan's observations of secondary minimum, which cover a space of more than two years. Figure 2 shows all of his individual observations between phases of twenty and forty hours from primary minimum plotted in three groups whose mean dates are separated by about a year. Unfortunately, the emergence is not so consistently well observed as the immergence, so that only the time of beginning of eclipse is useful here. Evidence for a progressive retardation of phase, which is in order at this epoch under the circumstances in question, is pretty weak, and in any case the shift can hardly be greater than that indicated by the inclined broken line—much too small for a uniform orbital rotation of the speed suggested.

c) The period.—Mrs. Shapley¹ has very completely assembled the observations upon the primary minimum of U Cephei up to 1914, and computed their residuals from Wendell's light-elements, concluding that, apart from a rather abrupt change in the period in 1905, the changes, while real, were slight and complex. In Table X are gathered residuals from observations made since Mrs. Shapley's paper, and an earlier one by Schwerd.² All data are plotted in Figure 3.

In a rotating orbit of slight eccentricity a plot of residuals from linear elements against time gives a sine-curve,<sup>3</sup> but when, as in this

TABLE X
RESIDUALS FROM WENDELL'S ELEMENTS

Epoch J.D.	O-C	Observer	Reference
2388855	od733:	Schwerd	Chandler, A.J., 9, 49
2421290	.0220	Dugan	Princeton Contr., No. 5
2421997	.0275	Campbell	Personal communication
2422337	.0297	Dugan	Princeton Contr., No. 5
2423054	.040	Campbell	H.B., No. 762, 1922
2423249	.047	Campbell	Personal communication
2423727	.065	Stetson	Personal communication
2424804	.0846	Campbell	H.B., No. 842, 1927
2425557	.0975	Campbell	Ibid., No. 862, 1928
2425901	0.101	Campbell	Ibid., No. 871, 1929

case, the eccentricity is much too large to be expanded usefully into a series, the resulting curve, having an amplitude here of eight or ten hours, is quite asymmetrical, although the actual curve is hardly computable, since, on account of the variation of the perturbative force throughout the orbit, the orbital rotation will not be uniform. It is at first tempting to interpret the plotted curve as a part of one resulting from orbital rotation, though the period would probably not fit well. A uniformly increasing period, however, gives a residual curve of the form of a vertical parabola, so that it appears more informative to deduce the true periods from the figure by computing

I Ibid

 $<sup>^2</sup>$  For a discussion of the justification of using Schwerd's observation see Dugan, op.  $\varepsilon it.$ , p. 29.

<sup>3</sup> André, Traité d'astronomie stellaire, 2, 253, Paris, 1900.

from the slope at any epoch the corresponding correction to Wendell's period. Figure 4, so derived, which is essentially an extension of Dugan's to later dates, shows that the complete history of the period, as far back as our observations extend, involves an essentially uniform increase with small superposed oscillations. With due

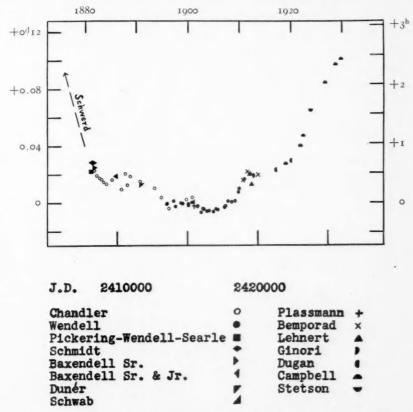


FIG. 3.—Residuals from Wendell's elements. As far as the hump about 1912 the curve is essentially the same as Mrs. Shapley's. The various observers are indicated by the symbols above. For the original references see Mrs. Shapley's and Dugan's papers, and Table X of this paper.

consideration to the inherent inaccuracies of graphical differentiation, it seems difficult to doubt the reality of those oscillations, since the corresponding oscillations in the residual curve from which they are derived are well above the presumable minute or two uncertainties<sup>1</sup> of observation. But their amplitude and period, especially the former, are much too small to fit any hypothesis of advance of periastron.

The definite reconciliation of the photometric and spectrographic data on the basis of a rotation of the orbit in its own plane, which was tentatively suggested as a possibility in a preliminary note,<sup>2</sup> has then to be abandoned in spite of the favorable representation of the light-curve on the assumption of a reasonable speed of rotation. It then becomes equally difficult to explain the apparent absence of such rotation in a system presumably so well suited to its detection.

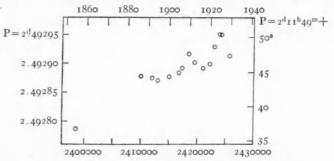


Fig. 4.—The period of U Cephei through eighty years

U Cephei appears to be a system much more complex than its idealized light-curve would imply. Granted Dugan's reasonable explanation of the uniform increase of period as due to tidal evolution, the oscillations of the period are still a mystery. G. Viola<sup>3</sup> recently, from mean light-curves made some years ago at Catania and Capodimonte, and H. T. Stetson<sup>4</sup> earlier from single light-curves, have shown a lack of constant light during the total phase of primary minimum. Unfortunately, the secondary minimum is so shallow as to be studied only with considerable difficulty. Probably the most effective line of attack upon these complexities at the present time would be close following spectrographically.

Steward Observatory University of Arizona August 1930

<sup>1</sup> Dugan, op. cit., p. 33.

<sup>&</sup>lt;sup>2</sup> Popular Astronomy, 38, 401, 1930. <sup>3</sup> Atti Lincei (6th ser.), 10, 508.

<sup>4</sup> Astrophysical Journal, 43, 325, 1916; Popular Astronomy, 32, 623, 1924.

## TEMPERATURE CLASSIFICATION OF THE SPECTRA OF EUROPIUM, GADOLINIUM, TERBIUM, DYS-PROSIUM, AND HOLMIUM, λ 3850 TO λ 4700<sup>1</sup>

#### By ARTHUR S. KING

#### ABSTRACT

Spectra of europium, gadolinium, terbium, dysprosium, and holmium, as given by electric furnace, arc, and spark, have been examined in the range  $\lambda$  3850 to  $\lambda$  4700 with regard to the segregation of neutral and enhanced lines, temperature classification, the occurrence of hyperfine structure, and other distinctive features. The total of 2277 lines listed is made up of 219 lines of europium, 627 of gadolinium, 733 of terbium, 534 of dysprosium, and 164 of holmium.

Measurements of wave-length were made for 666 lines. These are in part new lines prominent in the furnace but faint in the arc spectrum, partly known lines for which improved measurements could be obtained, and partly close doublets previously meas-

ured as single lines.

The spectra of gadolinium, terbium, and dysprosium are especially rich in neutral lines which are relatively strong in the furnace. Europium and dysprosium show low-temperature lines which have high intensity and are easily reversible, a type which is

unusual in the spectra of the rare earths.

Lines having hyperfine structure are numerous in the spectra of europium, terbium, and holmium. The patterns of hyperfine holmium lines resemble those of praseodymium. The present material, taken in connection with data at hand for other rare earths, shows that hyperfine structure occurs regularly in the spectra of rare earths of odd atomic number, as far as their spectra have been observed with reference to this phenomenon, while for even numbered rare earths the spectrum lines are uniformly without hyperfine structure.

The treatment of material in this paper is similar to that for the spectra of cerium and praseodymium,<sup>2</sup> the chief object being the temperature classification of the lines of both the neutral and the ionized atoms by means of a comparison of furnace spectra at several temperatures with the spectra of the arc and spark. Preliminary to this classification, it was necessary to distinguish between lines of the neutral and of the ionized spectra, and throughout the entire investigation attention has been paid to special features which may aid in an analysis of the spectra when these have been examined through a greater range of wave-length.

The spectral region examined is the most useful for comparison with celestial spectra, and, on account of the mingling of neutral

<sup>&</sup>lt;sup>1</sup> Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington, No. 414.

<sup>&</sup>lt;sup>2</sup> Mt. Wilson Contr., No. 368; Astrophysical Journal, 68, 194, 1928.

and ionized lines in about equal proportions, shows best the characteristics of the spectra concerned. For the rare-earth spectra thus far studied, ionized lines are relatively numerous in the violet, and at shorter wave-lengths make up almost the whole spectrum, while in the blue-green and farther to the red the neutral lines predominate.

Between  $\lambda$  3850 and  $\lambda$  4700, as for cerium and praseodymium, the ionized lines are strong in the arc spectrum, and normally exposed spectrograms of arc and spark thus appear much alike. The furnace spectrum has a very different appearance, being made up of the neutral lines, often inconspicuous in the arc, while the ionized lines which persist in the furnace are faint as compared with their arc intensities. A mixture of caesium with the element studied has been used in the furnace for all five of the elements. The large supply of free electrons obtained by reason of the low-ionization potential of caesium produces a recombination of the ionized rare-earth atoms and thus suppresses the ionized lines at temperatures for which many of them show distinctly when the rare earth is vaporized alone. The spectral characteristics of the individual elements will be considered later in the paper.

#### EXPERIMENTAL METHOD

The second order of the 15-foot concave grating was used, giving a dispersion of 1.86 A per millimeter. For the classification of the lines the carbon-tube furnace was operated in vacuum at temperatures of approximately 2000°, 2300°, and 2600°–2800° C for the low, medium, and high stages, respectively.

The samples of rare earths used were oxides, prepared by G. Urbain, of Paris, and obtained by the writer from Professor G. Eberhard, of Potsdam. The impurities present in some of the samples were chiefly other members of the group of five under investigation, and an intercomparison of spectra usually indicated clearly the source of the impurity lines.

All these elements belong to the scarcer group of rare earths, and to conserve the material the oxide was usually placed in a graphite combustion boat located midway in the furnace tube. This tended to avoid explosive expulsion of the oxide at the first heating. For the arc and spark, the oxide was either held in the lower electrode of carbon or graphite, or, when the elimination of the carbon spectrum was desirable, mixed with silver filings in a bored-out silver electrode.

#### MEASUREMENTS OF WAVE-LENGTH

Most of the existing wave-length tables for these elements leave much to be desired. The scarcity of the substances has resulted in measurements by only a few observers, for the most part before modern standards of wave-length were established. In the present work 666 lines were measured, chiefly in the furnace spectrum from iron standards or from good lines of titanium or vanadium given by impurities in the graphite tube. Part of these wave-lengths are for the stronger known lines, both neutral and enhanced, for which good three-place values could be obtained on account of their sharpness in the present spectrograms. Others are for neutral lines distinct in the furnace spectrum, though faint in the arc and not previously measured. A third group consists of close doublets, measured by former observers as single lines but clearly resolved on my plates. Such a pair often consists of a neutral and an ionized line, and these were measured in either the furnace or the arc spectrum, or in both, to obtain the best value for each component. Measurement of hyperfine lines was usually deferred until plates of higher dispersion had become available, since the components are often graduated in intensity, as well as in spacing, and a single wave-length for such a pattern is of limited value. Some of these wave-lengths, as well as those of some faint single lines, are given to only one decimal place.

The writer was assisted in the work of measurement and in making up the tables by Miss Brayton, of the Computing Division.

#### HYPERFINE STRUCTURE

In studying the spectra of lanthanum<sup>1</sup> and praseodymium<sup>2</sup> it was noted that a large proportion of the lines of these elements are complex. Such lines of lanthanum form close patterns of at least three components, while the praseodymium patterns are wider and, when resolved, show regularly six components. For some of these the spac-

<sup>1</sup> Mt. Wilson Contr., No. 326; Astrophysical Journal, 65, 86, 1927.

<sup>2</sup> Loc. cit.

ing increases toward shorter, and for others toward longer waves. The structure of praseodymium lines was examined further under high dispersion by the writer, and then more extensively by H. E. White<sup>1</sup> and by Gibbs and Gartlein.<sup>2</sup>

The fact that lanthanum and praseodymium have odd atomic numbers, while the even-numbered elements, cerium, neodymium, and samarium, were found by the writer to show no complex lines beyond occasional doublets, suggested that the frequent occurrence of hyperfine lines might be a characteristic of the odd-numbered rare earths. This is fully borne out by the present investigation, in which complex lines were found to be numerous in the spectra of europium (63), terbium (65), and holmium (67), while the lines of gadolinium (64) and dysprosium (66) show no complexity. With the exception of illinium (61), we have data concerning hyperfine structure for elements 57-67 inclusive; and, as an extension of the evidence, it may be added that my preliminary spectrograms of erbium (68) show no hyperfine lines, while a letter from Dr. Meggers states that the lines of lutecium (71) have hyperfine structure and those of ytterbium (70) are sharp. Complex line structure thus seems to be general for the odd-numbered rare earths. Although data are lacking for the spectra of illinium (61) and thulium (69), both may be expected to contain hyperfine lines.

Hyperfine structure, according to modern theory,<sup>3</sup> arises from the influence of nuclear spin on the outer part of the atom. The components of complex lines can then be expected to result from the slight changes of energy-level due to the magnetic moment of the nucleus. A consequence of the present observations would be that in the rare-earth atoms of odd atomic number the interaction between the nuclear moment and the outer electrons is much more pronounced than for the general run of elements, which only occasionally exhibit hyperfine structure. The hyperfine patterns of holmium and praseodymium are comparable in size and similar in appearance to those of bismuth, the element for which this phenomenon has been most carefully studied.

<sup>1</sup> Physical Review, 34, 1397, 1929.

<sup>&</sup>lt;sup>2</sup> Unpublished.

<sup>&</sup>lt;sup>3</sup> Pauling and Goudsmit, Structure of Line Spectra, 1930.

#### EXPLANATION OF THE TABLES

In Tables I to V the first column gives wave-lengths on the international system. The sources of these, when taken from lists already available, are noted in the descriptions of the several spectra. An asterisk (\*) refers to a note at the end of the table, and a dagger (†) indicates that the line was measured during this investigation.

The second and third columns give intensities for the arc and high-temperature furnace, respectively. The addition of R or r indicates complete or partial self-reversal. A minus sign after intensity r means that the line is very faint on the arc spectrogram examined, which was of such general intensity as not to show serious over-exposure of the stronger lines. The intensities at medium and low temperature, which were required for the classification, are omitted in order to condense the tables. The relative intensities of the lines at these lower temperatures are indicated in some degree by the class designations in the last column. Thus the more decided low-temperature lines are in class r, and those strengthening more rapidly with rising temperature in class r. Lines of class r are faint or absent at low temperature, but strong at the medium stage, while lines of class r appear only at high temperature and those of class r only in the arc.

For the spectra of europium, terbium, and holmium, Arabic numbers denoting the probable number of hyperfine components making up the given line are placed after the class numbers. The addition of v or r to the number indicating hyperfine structure means that the pattern in question has wider spacing of the violet or of the red components.

 $\begin{tabular}{ll} TABLE\ I \\ Temperature\ Classification\ of\ Europium\ Lines \\ \end{tabular}$ 

	INTE	NSITIES	CLASS,		INTE	NSITIES	CLASS,
λ (Ι.Α.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comps
3884.76	50	8	III, 3	3988.22	3		VE, 3
3891.46	1		V, 2	3988.55	1		V E, 2
3803.40	4	3	IV ?, 4	3992.36	2		V
3896.84	6	2	IV, 2	3993.91	1		VE
3897.80	10	3	IV, 2	3996.00	2		VE
3898.80	12	4	IV, 2	4000.72	3	2	IV, 2
3000.15	8		V E, 3	4000.80	2	I	IV, 2
3000.50	6		V E, 2	4003.74	2		VE
0, 0	I		V, 2	4010.42	4	2	IV. 2
3901.70	8	2	IV, 2	4011.71		13	VE
3903.18	-				25		VE
3907.113*†	300R	30	III E, 4	4012.83	3		
3909.89	3		V, 2	4014.39	4	2	IV, 2
3910.13	2		V, 2	4014.66	2	I	IV, 2
3914.14	4		V, 2	4016.72	10	6	III, 2
3915.24	3		VE, 2	4017.63	15		V E, 2
3915.62	6	2	IV, 2	4025.97	2		V, 2
3915.98	20	4	III, 2	4026.62	6	13	IV ?, 2
3016.80	4	2	IV, 2	0.6.	12	I	IV
3917.30	30	5	III, 3	4028.64	15	2	IV, 2
3917.68	4		V. 2	4030.04	20	8	III, 2
3918.48	8	4	IV, 2	4036.11	12	6	IV, 4
	6		V E, 2	4030.20	30	12	III, 2
919.12	2	13	IV?	4040.48			III, 3
927.46					15	5	V E, 2
3928.92	6		V, 2	4042.05	4		V L, Z
929.88	4		V, 2	4058.49	2		V, 2
930.425*†	15?	2?	III E	4059-47	6		VE, 2
930.504	300R	50	III E, 3	4062.23	3		VE, 2
935.88	2		VE	4065.48	4	2	IV
936.64	3	13	IV, 2	4066.05*	2?		VE
941.55	3		V E, 2	4069.02*	4	I	IV, 3
942.21	6		V, 2	4071.25	8	4	III, 2
942-35	4	1?	IV?	4078.23	7	3	III, 2
043.10	12		VE, 2	4085.36*	15	I	IV E, 2
949.57	10	3	III, 4	4087.83	5		VE
955-73	20	5	III, 4	4096.84	8		VE, 2
957.91	4		V E, 2	4102.72	3	2	IV
961.15	4	2	IV, 3	4103.87*	2	3	IV, 2
063.63	4	2	IV, 2	4106.87	15	6	III, 2
7 . 4	6		IV, 4		112		V É, 2
964.48		4	IV E, 2	4112.08	6		V E, 3
964.92	20	2	III, 3	1776.07	I		IV, 3
967.12	5	5	111, 3	4116.97			V E, 3
969.23	8	2?	IVE, 2	4119.35*	12		
971.899*†	20?	33	III E	4127.30	15	6	III, 2
971.989†	400R	60	III E, 3	4129.639*†	25?	4?	HIE
976.87	3	1	IV, 2	4129.7341	500R	80	III E, 3
978.45	20	3	III, 3	4136.62	2		V E, 2
979.08	4		V E, 3	4137.11*	105	5	III, 2
979.62	3		V E, 2	4141.03	5		V E, 2
985.35	2	I	IV, 2	4141.73	7		V E, 2
3 3 3 3		{3		4145.26	2		VE, 2
986.64	15		IV, 2				

TABLE I-Continued

	INTE	NSITIES	CLASS,		INTE	NSITIES	CLASS,
λ (Ι.Α.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comps
4151.56	6		V E, 2	4352.20	2		V E, 2
4157.81*	20	5	IV, 2	4354.82	20	4	III
172.81	10		V É, 3	4355.10	30		VE
1175.22	3		V E, 2	4357.71	I		VE
176.66	2	2	III, 2	4360.04	I		V, 2
182.28*	20	5	III, 2	4361.57	I		VE
194.46	2	2	IV, 2	4368.43	1		V E, 3
195.40	3		V E, 2	4369.46	6		V E, 3
202.05	5		V E, 2	4370.43	8	3	III, 2
1202.65	8	4	III, 3	4372.22	I		VE
204.909*†	30?	5?	III'E	4375.14	I		VE
1205.046†	600R	100	III E, 3	4383.02	15		V E, 2
					20	10	III, 2
1209.09	4	2	III, 3	4387.85			
220.71	3	2	IV, 2 V E	4391.35	3		VE, 3
1221.10	1			4397.73	I		
1222.33	5	3	IV, 2	4399.34	1		V, 2
1224.30	3		V	4405.28	3	I	IV, 2
1229.34	3		V E, 2	4407.07	2		VE, 2
1232.48	5		V E, 2	4413.55*	2	I	IV, 3
234.13	2		VE	4417.25	8	4	III, 3
235.14	2		V, 2	4419.65	1		VE, 2
237.52	4		V E, 2	4426.41	I		V E, 2
238.72	2		V E, 2	4429.76	1		VE
244.79	10	6	III, 2	4434.81	8		VE
247.07	3		V E, 3	4435-473	20?	10	HE
1249.49	3		V, 4	4435.602	400r	80	III E, 3
253.85	2		VE	4464.94	10		VE.
255.27	2	I	IV, 2	4471.99	3	2	IV, 2
258.13	4	3	IV, 3	4477.26	2		V, 2
269.48	I		V, 2	4485.17	6		VE, 2
270.52	2		VE	4488.30	2		V E, 2
276.16	3		V E, 2	4516.55	2		V, 3
281.90	I		V E	4517.36	1		v, s
284.70	2	13	IV, 3	4522.602*†	200r	50	III E, 4
287.45	I	1.	V , 3	4526.69	6	3	III, 2
288.60			V E, 2	4535-59*	15	82	III, 2
	I				-	0.	
.292.96	2		V, 3 V E	4539.29 4586.46*	2		V E, 2
293.92	2		VE		3 P	and D	V, 3
295.43	1			4593-993†	500R	200R	II
298.73*	30	4?	III	4597.35*	5	2	IV, 3
300.84	1		VE	4616.53	3	3	IV, 3
317.67	I		VE	4625.32	6	3?	III, 2
322.58	6	4	III, 3	4627.1221	300R	150R	H
329.38	20	8	III, 3	4641.40	I		V
330.00*	20	3	III?, 2	4644.25	4		V
330.62	15		VE	4650.47	6	2?	IV, 2
331.20	8	4	III, 2	4651.56	1		V
334.16	I		V	4656.70	8	3	III, 2
334-74	I		VE	4658.60	4	3	IV, 2
337.67	25	8	III, 2	4660.35	10	3?	III, 2
343.26	4	2	IV, 2	4661.865†	250R	100R	II
345.91	12	5	III, 2	4665.07	3		V, 2
349.78	I	3	V, 3	4671.20	2	13	IV, 2
079.10	*		VE	4675.46	-	3	IV, 2

#### NOTES TO TABLE I

λ

3907.113 Line measured is unresolved triplet; faint fourth component to violet not measurable

3930.425 Not fully resolved from strong triplet

3971.899 Not fully resolved from strong triplet

4066.05 Blend Gd I

4069.02 Violet component stronger

4085.36 Red component shows in furnace

4103.87 Furnace line probably Ho

4119.35 Components partially resolved

4129.639 Not fully resolved from strong triplet

4137.11 Blend Gd II in arc

4157.81 Blend Gd 1 in furnace

4182.28 Furnace line largely C

4204.909 Not fully resolved from strong triplet

4298.73 Close to Ti in furnace

4330.00 Blend V in furnace

4413.55 Components partially resolved

4522.602 Line measured is unresolved triplet; faint line to violet not measurable

4535.59 Blend Ti in furnace

4586.46 Very diffuse

4597.35 Shaded to violet

TABLE II
TEMPERATURE CLASSIFICATION OF GADOLINIUM LINES

	INTE	NSITIES			INTE	NSITIES	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
836.90	100	2?	IV E	3935-393†	40	30	II
839.63	60	I	IV E	3938.13	8		VE
840.264	6	5	II	3938.98	12		VE
842.20	80	1	IV E	3041.802†	40	25	II
843.275†	40	10	I	3042.643†	60	30	I
844.58	125	2	ÎVE	3943.244	30	20	II
		3	IVE	3943.631†	30	20	III
850.69	150		HE	3945.548*†		100	I
851.00	200	4 2	IVE		150	8	ÎII A
852.50	150	1		3945-7531		6	IV
855.57	20		VE	3947.05	6	0	-
863.05	10		VE	3949.23	8		VE
866.989†	30	15?	I	3952.01	40	I	VE
867.62†	8	6?	II	3953-372†	80	40	I
874.476†	8	6	II	3957.69	150	2	IV E
875.33	10		VE	3958.681†	8	30	III A
884.71	4		VE	3959-437†	30		VE
887.16	10		VE	3959.526†	50		VE
887.739†	8	15	III A	3060.113†	15	30	III A
888.94	12	5	III	3062.12	6		VE
800.415†	8	10	IV	3963.66	15		VE
			VE	3964.97*†	13	1	IV
890.88	4		IV			1	III
892.75	4	4		3965.03*†	3	4	II
894.72	200	6	IVE	3966.279†	60	60	-
895.25	20		VE	3966.84	4		VE
895.7917	40		VE	3968.34	15		VE
895.960†	4	8	IV A	3969.004†	60	50	Ι.
896.45	4		VE	3969.28	40		VE
897.323†	8	8	IV	3970.199†	4	5	IV
898.46	4		VE	3971.09	20		VE
902.42	150	4	IV E	3971.77	40	I	VE
902.718†	30	25	II	3972.20	8		VE
904.293†	20	25	II	3972.713†	30	40	I
905.653†	30	25	II	3974.06	40	1	IV E
2 0 00.	6	1	IV	3974.241†	2		VE
907.12	2	5	VE	3974.819†	30	40	II
908.149†			IV	39/4.019	30	10	VE
909.25	4	8	III	3975.13	8	2	IV
909.945†	6				60	80	IIA
910.23	2	,	VE	3979.346*†	60		
911.66	4	5	IV	3979-759†	2	2	IV
912.76	8	10	IV	3980.14	2	2	IV
913.79	6		VE	3983.04	15		VE
016.58	300	3	IV E	3984.82	2	6	III A
016.672	10	12	IV	3985.67	2	I	IV
018.07	15		VE	3987.24	60	2	IV E
918.26	15		VE	3987.840†	50	50	II
923.249†	30	I	IVE	3989.27	8		VE
923.333	8		VE	3001.70	2		VE
	2	1	IV	3992.696†	20	40	IIA
924.23			IV	3992.0901	20	I	IVE
926.688†	12	15	VE			I	IVE
932.95	15			3994.19	40	1	VE
934.790*†	200	50	I	3996.34	40	******	V E

TABLE II—Continued

	Inti	ENSITIES			Inti	ENSITIES	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
3997.78	20		VE	4040.107†	15	40	III A
1000.16	4	10	IV A	4049.44	150	I	IV E
4001.20	30	tr	IVE	4049.548†	4	8	III A
4001.63	2	2	IV	4049.90	200	2	IV E
4001.95	4		VE	4050.373*†	10	20	III A
4003.37	4	6	IV	4051.62	2	6	IV
4003.85	6		VE	4053.31	100	-	VE
1004.93	20		VE	4053.643†	200	150	I
1006.975†	6	30	III A	4054.731†	80		Î
1008.339		-	IIA			75	VE
	25	50	VE	4056.04	2	200	I
4008.91*	30	1	VE	4058.222†	250	200	VE
4009.21	8			4059.39	6		
1013.430†	6	8	III	4059.876†	40	50	III
4013.83	20	******	VE	4060.933†	1	3	HIIA
4013.92	8		VE	4061.163†	4	20	III A
4015.21	6	6	III	4061.30	10		VE
1015.592	6	10	III	4061.85	4	8	III A
1017.255	10	8	III	4062.61	50		VE
1017.720	20	12	III	4063.45	150	I	IV E
1019.739	12	40	II A	4065.63	2		VE?
1020.42	2	I	IV	4066.11	10	20	III A
021.91	2	2	IV	4068.358†	15	30	III A
1022.33	15		VE	4068.77	10	20	III A
023.154†	80	100	I	4070.202†	40	I	IV E
023.355†	60	100	I	4070.396†	20	1	IVE
1023.76	2	3	IV	4073.21	40	I	IVE
027.613†	12	50	III A	4073.78	200	2	IVE
028.155†	40	50	III	4075.47	4		VE
030.881†	30	50	III	4078.47	150	2	IVE
032.11	2	3	IV	4078.705†	300	200	Î
033.493†	30	50	III	4080.534†	15	30	ÎI A
	2	1 -	IV		-	1	IV
034.05		I	III A	4080.78	4	3	III
034.38	2	4	IIA	4083.710†	30	50	VE
035.396†	20	60	III A	4083.95	2		IV
.036.842†	15	40		4084.35	2	1	
037.34	200	2	IVE	4084.71	4		VE
037.90	125	I	IV E	4085.60	200	3	IVE
038.28	2	3	IV	4087.330	2	8	III A
039.50	10		VE	4087.71	20		VE
039.67	15		VE	4087.847†	2	10	III A
042.50	2	2	IV	4088.81	2		VE
042.77	4		VE	4090.418†	40	30	I
043.712†	15	40	III A	4090.747†	2	3	III
044.030†	4	20	III A	4091.754†	4	8	HIA
045.013	60	100	I	4091.966†	2	3	III
045.11	12		VE	4092.718*†	100	80?	I
045.862*†	3	30	III	4093.723†	15	30	III A
046.84	15		VE	4094.48	30	I	IVE
047.087†	12	40	III A	4008.04	4		VE
047.81	8	4-	VE	4098.64	300	4	IVE
048.59	8		VE	4098.90	80	I	IVE
048.82		8	III A	4100.269†	60	50	I
140.02	4	0	TITLE	4100.209	00	20	A

TABLE II-Continued

λ (Ι.Α.)	Intensities				Intensities		C
	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
104.99	8	12	III	4188.77†	2	1	IV
105.79	2		VE	4189.10	2	I	IV
108.42	6		VE	4190.16	6	10	III
1100.493†	2	10	III A	4190.779†	200	200	I
	6		VE	4191.06	200	200	ÎV E
110.432†			VE			100	I
110.609 †	20		HIA	4191.628†	125		VE
111.251†	2	3		4193.14	6		
111.45	60		VE	4197.06	15		VE
111.74	12		VE	4197.68	80	I	IV E
112.93	6	8	III	4200.28	2	2	IV
113.78	2		VE	4202.49	8		VE
115.37	8		VE	4204.839†	100	I	VE
119.199†	6	4	III	4206.76	2	3	III
119.380	12		VE	4208.064†	6	6	III
121.05	2	2	III	4212.01	200	2	IV E
123.00	2		VE	4214.97	150	. 3	IV E
123.40	2	I	IV	4216.49	2	2	IV
123.59†	2	4	III A	4217.15	100	I	IVE
125.76	4	5	IV	4222.580†	2	4	III A
			IV A	4222.98			VE
127.3†	2	4	VE		4	10	III A
127.68	4		IV	4224.22	4		IIA
129.2†	2	3	VE	4225.028†	20	50	
130.35	300	3		4225.156†	15		VE
131.48	40		VE	4225.264	4	3	III
132.28	200	2	IV E	4225.846†	300	300F	I
.134.169†	100	100	I	4227.14	20		VE
137.09	50		VE	4229.77	15		VE.
138.020	2	5	III A	4232.46	4		VE
140.450	12		VE	4232.941	4	3	IV
140.567†	2	3	III	4235.03	2		VE
141.00	2		V E	4235.890	8		VE
144.240	6	8	III	4238.77	80	1	VE
148.864*†	20	40	II A	4240.67	2	6	III A
149.475†	8	10	III	4241.30	8		VE
150.61	2		VE	4243.85	15		VE
151.59	2		VE	4245-353†	12	25	III A
153.53	12		VE	4246.52	15		VE
154.86	20		VE	4250.281†	4	10	III A
157.781†		50	IIA	4251.76*	300	6	IV E
	15		III		-		IVE
158.469†	60	2	VE	4253.36	150	I	VE
162.72	60		III	4253.62	150	8	III A
163.111*†	30	15		4254.03	4		
167.157	6	******	VE	4255-44	4		V
167.271†	12	30	II A	4255-54	4	******	
170.11*	10		VE	4260.123†	125	100	II
171.72	8	8	III ·	4262.004*†	250	40	III
173.56	10		VE		-	4-	VE
175.539 1	200	150	I	4266.234	2	4	III A
182.76	4	8	III A	4266.601†	125	100	II
183.65 †	2	2	IV	4267.0101	80	80	II
184.25	300	3	IV E	4268.73	30		VE
	10		VE	4270.26	6	10	III

TABLE II—Continued

λ (Ι.Α.)	INTE	Intensities			INTE	Constant	
	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
270.80	2	4	III A	4336.50	2	2	IV
273.25	4	8	III A	4336.71	2	2	IV
1274.167	40	60	II	4337.504*†	15	20	III
1278.20	2		VE	4340.25	2	2	IV
280.50	200	5	IVE	4340.47*	2	I	IV
282.78	4		VE	4341.9†	2	2	IV
1284.953†	4	8	III A	4341.24	150	2	IV E
1285.483†	2	5	III A	4342.18	300	3	IV E
285.8227	100	40	III	4344-34	20		VE
286.127	100	60	II	4344-45	4		VE
280.01	8	00	VE	4346.460†	400	250	I
	6	10	III	4346.624†	200	150	Î
1290.065†		10	VE			130	VE
292.76	2	2	IV	4347.25	40	2	III
294.76	2		IVE	4353-74		3	VE
1296.04	150	2	VE	4354.02	4	8	III A
296.30	40		IV	4357.80	2	1	VE
296.509†	2	3	-	4359.14	4		VE
296.90	6	15	III A	4359.61	2		
297.15	40		VE	4360.12	2	3	IV IV E
.298.39	2		VE	4360.90	40	I	
299.301†	80	40	III	4364.13	2		VE
303.46	2		VE	4366.51	2	5	IV A
304.90	30		VE	4368.53	2	I	IV
306.348†	200	150	I	4369.161†	20	20	III
308.24	6		VE	4369.77	80	3	IVE
308.392	2	3	III	4370.183†	30	50	III
309.295	60	50	III	4372.01	2	I	IV
309.93	2	I	IV	4373.831	300	150	I
310.95	30		VE	4374-24	4		VE
313.851	200	200	I	4374.986†	4	4	IV
314.400	80	60	III	4376.083†	8	20	III A
316.05	150	1	IV E	4378.53	30	30	III
316.23	20	1	VE	4379.52	2	1	IV
320.521	80	60	III	4380.63	20		VE
321.1117	60		VE	4382.04	15		VE
321.202	100	100	II	4383.16	30		VE
322.012	2	5	III A	4386.18	6	3	IV
322.18	20	I	IV E	4387.15	6	3	IV
323.60*	4	3	IV	4387.63	100	2	IV E
324.04	20	I	IVE	4388.90	2		VE
324.57	4		VE	4389.893†	30	20	III
325.568†	30		VE	4390.003†	10	5.	5
325.691	200	200	I	4390.05	80	1	IV E
326.29	4		VE	4391.45	15		VE
327.106*†	250	200	I	4391.652†	2	3	IV
328.943†	12	25	ÎII A	4392.063	100	50	III
320.585	60	50	III	4392.77	2	1	IV
	2	30	VE	4394.71	4		VE
330.27		2	IVE	4397.388†	2	3	IV
330.58	100		III		60	3	VE
331.385†	40	40	III	4397.50*			VE
333.20	6	4	VE	4400.17	4	20	III
335.27	2		A T7	4400.765	15	20	TTT

TABLE II-Continued

> (F.A.)	INTE	NSITIES	CLASS λ (I.A.)	Intensities		CLASS	
λ (Ι.Α.)	Arc	Fur.		A (I.A.)	Arc	Fur.	CLASS
401.849†	300	150	I	4465.78	6	12	III A
403.129	100	40	III	4466.547†	* 0 #	13	VE
403.224	8	4	IV	4466.604†∫	125	15	III
406.68*	60	3	IV E	4467.088†	150	40	III
407.19	2	3	IV	4467.17	20		VE
408.25*	80	3 ?	IVE	4468.18	2		VE
409.264	40	30	III	4470.45	2	I	IV
411.160	150	80	I	4471.28	60	I	VE
413.44*	4	4	III	4473.282†	20	40	IIA
414.162	200	60	III	4474-131†	150	40	III
414.741	200	100	I	4475.08	2	2	IV
415.029	6	10	III	4476.144†	300	150	I
415.42	10	5	IV	4478.79	80	I	IV E
415.63	2	6	III A	4479.834†	2	3	III
415.97	2	2	IV	4481.06	100	2	IV E
419.03	150	2	IV E	4483.32	80	1	IV E
420.64	2	1	IV	4484.49	2		VE
421.21	30		VE	4484.710†	30	15	III
422.409	300	150	I	4485.484†	8	30	II A
424.00	4		VE	4486.34	12		VE
425.01	15	10	III	4486.908†	125	100	II
426.13	12		VE	4488.44	12		VE
427.58	10		VE	4488.57	8	1,2	III
428.93	2	6	III A	4489.77	2	3	III
430.631	300	150	I	4491.27	2	6	III
431.769	15	20	III	4492.03	2	2	IV
433.65	8		VE	4494.84	4		VE
436.103*†	20	30?	III5	4496.61	8	12	III .
436.226†	40	I	IVE	4497.133†	150	100	I
437.84*	6	3	IV?	4497-33	40	15	III
438.143†	4		VE	4497.55	2	I	IV
438.262†	60	3	IV E	4498.26	60	1	IVE
438.463†	4	4	IV	4499.89	2	6	III A
444.95	4	4	IV	4500.65	2		VE
445.53	2	1	IV	4500.89	2		VE
446.46	30	I	IVE	4501.49	2	2	IV
447-34	2	I	IV	4502.30	2	4	III A
448.13	2	8	IV A	4503.17	8	5	III
449.01	4	3	IV	4503.8031	8	30	II A
449.36	6	4	IV	4504.96	4	10	III A
449.95	2		VE	4506.226†	200	125	I
452.71	8	12	IV	4506.337†	60	I	IV E
453.92	6		VE	4506.933†	8		VE
454.71	4	3	III	4507.065	4	8	III A
456.67	2	I	IV	4507.66	12	10	III
	S2	1	IV	4508.03	2	I	IV
458.35	12	I	IV	4508.986†	10	8	III
461.079†	2	I	IV	4509.079†	6		VE
461.35	6	10	III	4510.38*	4		VE
462.78	20	12	III	4512.43	2	5	III A
463.23	12		VE	4513.80	4	5 8	III A
							VE

TABLE II—Continued

	Inti	Intensities			Inti	Cana	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
4516.980†	10	6	III	4582.49	40		VE
4517.075	2.		VE	4583.083†	125	80	III
4518.76	2	6	III A	4583.37	2		VE
4519.661†	200	150	I	4584.27	8	15	III A
4520.069	15		VE	4585.16	2	2	IV
4520.77	2	4	IV A	4586.07	4	3	IV
4521.291	12		VE	4586.97	25	15	III
4521.919†	15		VE	4588.72	2		VE
1522.43	2	2	IV	4589.54	2	8	IV A
1522.82*	80	1 20	III	4590.00	2	I	IV
1522.82	80	1 ?	VE	4594.28	2	I	IV
523.848†	2		VE	4595-37	4	6	III
1524.00	15	15	III	4596.98	60	I	IV E
524.33	2		VE	4597.91*	100	4	IV E
526.01	2	4	IV A	4598.897†	100	80	III
531.12	6	12	III A	4601.03	100	I	IV E
531.79	10	15	III	4602.932†	40	60	II
533.49	6	5 ?	IV	4606.06	8	2	IV
534.74*	2	5	3	4606.64	2		VE
536.96	15	15	III	4608.00	4		VE
537.8201	200	150	I	4608.583†	8	30	III A
539.18	2	2	IV	4611.04	4	2	IV
540.01	20		VE	4611.60	2	1	IV
541.22	2	I	IV	4614.498†	150	100	III
542.034	125	100	II	4619.14	10	5	III
542.72	8	15	III A	4622.187†	2	4	III A
543.60	2	4	IV A	4622.311†	4	7	III
544.23	8	20	III A	4624.427†	20	40	III A
545.11	I	3	IV A	4627.62	6		VE
548.011	80	50	III	4630.50	6	4	III
549.47	2	6	III A	4636.649†	100	80	III
550.15	1	3	III A	4637.27	4	8	III A
550.95	15		VE	4638.61	4	I	IV
551.45	2		VE	4638.98	15		VE
554.98	6		VE	4640.04	15	10	III
558.07	40	1	IVE	4640.543†	4	20	III A
559.61	2	I	IV	4641.30	4	3	IV
561.05*	8	6	IV	4645.98	20	8	III
563.40	2		VE	4646.33	6		VE
563.71	2	I	IV	4647.650	30	50	II
564.57	4	5	IV	4648.585†	8	3	III
570.838†	2	4	IV	4648.704	10	6	III
570.94	2		VE	4652.32	10	4	III
572.210	8	25	III A	4653.543	80	60	III
572.42	2	6	III A	4654.765†	4	20	HIA
573.811	40	80	II A	4654.98	10		VE
575.912†	60	60	III	4658.60	6	40	II A
579-593†	60	40	III	4659.84	2	10	III A
581.05	15		VE	4664.24	4	******	VE
581.294	100	100	II	4666.45	4	2112211	VE
582.332	2	6	III A	4668.23*	2	3	IV?
82.37	20		VE	4670.84*	15	8	III

TABLE II—Continued

λ (Ι.Α. )	Intensities		Crana		Intensities		CLASS
	Arc	Fur.	CLASS	λ (I.A.)	Arc	Fur.	CLASS
.676.98	4		VE	4679.16 4680.04	40	10	III
676.98 677.63 678.24	6	30	II A IV	4680.04	40	20	III

	NOTES TO TABLE II
λ	
3934.790	Double, red component is $Gd$ II; $\lambda$ for neutral line in furnace
3945.548	May be double
3964.97	Not fully resolved
3965.03	
3979.346	Double
4008.91	Blend Ti in furnace
4045.862	Blend $Fe$ in arc; measured in furnace without $Fe$
4050.373	Faint Gd I to violet
4092.718	Furnace line partly $V$ ; $\lambda$ measured in arc
4148.864	Faint neutral line on violet side
4163.111	Strong in spark, indicating blend with Gd II line; arc line measures 4163.084
4170.11	Probably double
4251.76	May be blend with $Gd$ I
4262.094	Close blend; violet component is <i>Gd</i> I
4323.60	Double
4327.106	Close doublet in arc; red component is Gd II
4337 - 504	Strong in spark; probably blend Gd II
4340.47	Probably blend Gd II
4397 - 50	Faint Gd I line to violet
4406.68	Blend V in furnace
4408.25	Blend V in furnace
4413.44	Double; red component is Gd 11
4436.103	Furnace line may be partly V
4437.84	Blend V in furnace
4510.38	Double
4522.82	Close blend Gd 1 and Gd 11; furnace line measured \(\lambda\) 4522.795
4534 - 74	Furnace line chiefly Ti
4561.05	Arc line probably double
4597.91	Probably blend Gd I and Gd II
4668.23	Furnace line in carbon band
4670.84	May be double

TABLE III
TEMPERATURE CLASSIFICATION OF TERBIUM LINES

λ (Ι.Α.)	INTI	ENSITIES	CLASS, No. COMPS. λ (I.A.)	Inti	ENSITIES	CLASS,	
	Arc	Fur.		A (1.A.)	Arc	Fur.	No. Comps
3837.18	5	3	III	3897.25	4	5	III
3841.77	5		VE, 4	3897.39	4		VE, 2
3842.49	40		VE, 6	3897.8521		J3	III
3845.61	10		VE, 4	3897.925	15	16	III
3847.88*	8	3	III ?	3898.734†	4	6	III
3848.75	100	6	IV E, 6	3899.19	200	6	IVE, 3
		2	III			0	VE, 3
849.59	4	2		3899.54	15		VE
851.86	5		VE	3900.40	2		
854.04*	2	3	III ?	3900.72	4	5	III, 2
855.38	10	15	II	3901.35	50	50	II
855.58*	10	5	V E, 4	3901.60	6		VE
866.54	6	6	III	3901.98	15		VE, 4
868.90	4		VE	3902.35	10		VE
869.75	15		VE, 3	3903.11	3	5	III
872.10*	2	-3	V ?	3904.186†	4	6	III
873.00	io	12	III	3904.65	ī		V
	8	12	VE	02 1 0		8	III
873.78		*****		3905.61	10		
874.18	200	10	IV E, 6	3905.83	2	5	III A
874.73*	5	4?	III ?, 2	3906.53	4		V E, 3
875.21	20		V E, 4	3907.65	3		VE
876.13	8-	8	III	3907.79	3		VE
876.47*	3	3	III ?	3907.9127	5	10	III A
876.67	8	6	III	3908.076†	20	30	II
877.56*	6	3	IV?	3908.66	4	I	III
878.21*	10		VE?	3909.150†	15	12	III
870.00*	4	4	III ?	3909.54	15	5	III
00 +		?	III?		-		VE
880.35*	8	8		3910.13	4	******	III
881.29*		0	III?	3910.40	5	5	VE
881.75*	4		VE?	3910.57	2		
885.12†	8	10	III	3910.85	5	8	III
886.01	3		VE	3912.25	5	10	III A
886.70†	3	5	III	3912.78	5		VE
886.83	20		VE, 4		0	16	III
887.201	2	8	HIA	3913.45	8	2	III
887.49	4		VE	3913-79*	2	I	III
887.67	25		V E, 4	3914.59	5	8	III
887.88†	10	8	III		2	2	III
				3914-73			
888.21	30	25	III, 3	3915.45	30	100	III A, 2
889.85	10	******	VE, 3	3916.64	4	8	III A
890.95	5	I	III	3916.94	4	4	III
891.76	2	3	III	3917.32	6	6	III
893.35	15		VE	3918.82	8		VE
893.70	5	4	III	3919.00	6	8	III
894.63	20	20	III	3919.54	40	I	IV E, 3
895.065†	4	5	III	3020.72	10		VE, 2
395.376†	3	2	III	3920.962†	I	4	III A
	3	2	III A	3921.264		8	III A
895.478†		1	III		4		III. 2
895.969†	3	5		3921.76	3	4	
896.04	25		VE	3922.09	20		V E, 2
896.488†	4	8	III A	3922.74	50	******	VE
896.60	25		V E, 2	3923-33	6		VE ·

TABLE III—Continued

λ (I.A.)	Inti	ENSITIES	CLASS, No. COMPS.	λ (I.A.)	Inti	ENSITIES	CLASS,
. (-1111)	Arc	Fur.			Arc	Fur.	No. Comp
3924.40	4		VE	3957-368†	12	20	II
3924.81	10		VE	3957-97	40		V E, 4
3925.45	150		V E, 3	3958.35	60		V E, 6
3926.09	1		VE	3958.62	8	3	III
3927.15	4		V E, 3	3960.10	6		VE
3927-35	2	1	III	3960.308†	5	10	III A
928.99	3	5	III	3960.69	8	8	III
3929.42	2	2	III	3962.634†	10	15	III
3929.75	5	2	III	3965.120†	15	10	III
3929.88	10	12	III	3965.95	15		VE
1930.76	8		V E, 4	3966.283†	12	8	III
932.366†	12	20	III	3967.21	15	I	IV, 3
932.55	3	3	III	3967.672†	20	10	III
933.469	6	3	III	3969.90	5	6	III
933.905	4	5	III	3970.19	30		VE, 4
934.40	10	4	III	3971.790†	30	20	III
935.25	50	I	IV E, 3	3972.05	20		VE
937.15	5	5	III	3972.93	4		VE
937.636†	15	20	III	3974.30	15		VE, 3
938.04	4	8	III A	3974.70	12	10	III
938.16	3		VE	3976.86	250	15	IVE, 6
939.60	200	4	IVE, 6	3977.7601	2	5	III A
940.10	2		VE	3980.28	3		VE
941.16	15		VE	3981.15	20		VE, 3
941.35	4		VE	3981.92	150		VE, 4
042.20	15		VE	3983.85	15		VE
942.94	6	10	III	3984.04	15		VE .
943.66	8	15?	III A	3984.84	6		VE
944.20	6	4	III	00	55		VE
946.87	150	3	IV E, 4	3985.08	15	5	III
947.25	5	3	III	3986.34	15		VE
947.51	2	I	III	3986.940†	3	8	III A
948.35	20		VE, 2	3987.69	8	20	III A
949-39	4	I	III	3989.50	6	20	III A
049.51	6		VE	3990.22	4	4	III
949.88	4		VE	3990.63	25	12	III, 2
950.147†	15	15	II	0,,	54	3	III
950.445†···	15	10	III	3991.59	14	5	III
50.76	4		VE	3992.20	2		VE
51.71	3		VE	3993.54	10	3	III
51.889†	8	15	III A	3995.15	8	15	III A
52.047†	8	6	III	3995.80	6	-3	V E, 2
52.60	2	1	III, 2	3996.68	2		VE
53.00	3	3	III	3998.41	15		VE
54.040†	10	6	III	3998.887†	5	20	III A
54.131†	5	8	III	3999.41	25		VE
054-51	1	2	III A	4000.01	10		VE
54.77	I		VE	4001.279†	15	20	III
55.05	2	2	III	4002.20	50	20	V E, 6
55.65	3	2	III	4002.60	100	2	IVE, 4
55.80	2	1	III, 2	4003.78	15	-	VE, 4
56.16	8		VE	4003.91	13		VE

#### ARTHUR S. KING

TABLE III—Continued

λ (Ι.Α.)	INTE	NSITIES	CTASS	CLASS,	INTE	CLASS,	
	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comp
4004.52	8		VE	4041.537†	3	8	III A
4004.618†	5	8	III	4041.846†	4	8	III A
4005.55	200	4	IV E, 4	4042.36	6		VE
4005.97	8	1	VE	4045.35	5	10	III A
1007.73	2	5	III A	4047.18	6	15	III A
1009.193†	10	10	III			13	III A
	12	10	VE	4048.80	6	2	III A
1009.54			II	1057 50		,	V E, 2
1010.064†	15	15	III	4051.52	4		VE,
1010.644	2	3		4051.87	30		
1010.741	6	8	II	4052.43	4		V E, 2
1010.869†	8	10	II	4052.87	20		VE, 3
1012.46	4		VE	4053.35	3	8	III A
1012.84	40	I	IV E, 4	4054.03	10	15	III A
1013.276*†	25	20	III	4054.14	25	15	III A
1015.52	15		VE, 6	4055-34	8	8	III, 3
015.625	4	10	III A	4056.10	2	2	III
015.04	8		V E, 3	4057.06	4		VE
016.04	2		VE	4059-47	4		VE
016.34	12		V E, 6	4060.30	40	30	II
016.86		4	III, 2	4060.86	40		VE, 6
	4		VE	4061.58	60	60	II ,
017.85	4 8		III			8	III A
018.48		5		4062.20	4	0	
019.14	20		V E, 4	4062.80	10		V E, 3
020.47	30		VE	4063.95	15		VE, 2
020.734	4	6	III	4066.21	30		VE, 4
021.13	4		VE, 2	4067.35	4	15	III A
022.870	10	12	III	4069.30	1		VE
023.716†	10	20	III A	4070.13	10	10	III
.024.10	30		VE, 6	4070.58	10		VE, 4
024.703 †	5	10	III A	4070.70	5	4	III
024.781	10	15	II	4071.21	10	15	III
025.146†	4	10	III A	4072.35	4		VE, 3
025.74	10		VE, 3	4072.71	10	15	III
026.39	3	I	III	4073.96	10	20	III A
028.28	15	1	VE	4074.19	8	8	III
028.59	6	6	III	40/4.19	120		VE, 4
**	2		III A	4075.22	15	20	III
029.25		5		1077.00	2		VE
031.65	50		VE, 4	4075.90			III A
032.282†	60	40	II	4079.15	2	5	
032.625	8	20	III A	4081.24	30	60	II A, 4
032.705	5	15	III A	4082.23	4		VE
033.05	200	6	IV E, 4	4082.79	15	15	III
036.219	12	10	III	4083.20	15	15	III
036.45	6		VE	4084.25	12	20	III, 2
038.864	10	15	III	4084.83	10	15	III
030.20	3		VE	4086.60	20	50	II A
039.482†	10	15	III	4088.65	4	8	III A
039.724	1	3	III A		103	15	III
	6	10	III	4089.34	(15?		V E, 3
040.100†		10	V E, 2	4089.50	6		VE
040.43	6	******	III A		2	2	III
040.677†	3	10		4090.14		8	III A
040.938†	2	5	III A	4091.32	4	0	III A

TABLE III—Continued

	INTE	ENSITIES	CLASS,		INTE	ENSITIES	CLASS,
λ (Ι.Α.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comps
4001.42†	4	4	III	4133.51	2	4	III A
4092.19	20	60	II A	4135-37	20	30	III, 3
4093.37	2	I	III	4136.45	4	5	III
4094.05	8	8	III	4139.06	10	30	III A
4094.03	15		V E,3	4139.35	3	I	IV
4094.45	15	40	II A	4139.78	20	60	III A
1005.03	10	10	III	4140.75	10	00	V E, 4
4095.93	2	2	III	4141.55	30	60	IIIA
4096.38		2	VE		6	12	III A
4097.44	10			4142.45	8		III
4098.85	6		VE	4143.24		12	II
4099.15	4		VE	4143.51	40	40	
4099.47	15		V E, 4	4143.63†	20	20	III
4100.94	8	8	III	4144-47	100	6	IV E, 4
4101.65	15	20	III, 3	4146.91†	15	40	III A
4102.54	15		V E,3	4149.16	6		VE
4103.21†	5	8	III	4150.87	4	10	III A
4103.37†	15	15	III	4152.24	5	15	III A
4103.49†	6	10	III	4153.49	5	I	III
1103.91	20		V E, 3	4153.84	3	2	III
1105.37	25	50	II A	4154.64	5	5	III ·
	-	6	III	4155.40	4	6	III
4106.35	4	8	III A	4156.28	8		III, 2
1107.77	4		III A		6	5	VE
1108.40	3	6		4158.28			II
1109.08	1	I	III	4158.54	30	50	
4110.09	2	6	III A	4159.19	3	2	III
4110.86	4	4	III	4160.01	1	2	III A
1111.00	2	2	III	4161.35	5	8	III ·
1112.53	30	40	III	4162.99	3	2	III
4112.88	15	20	III	4167.12	4	8	III A
1114.15	20		V E, 6	4169.11	10	15	III
1115.34	20		VE, 6	4169.30	20	30	III
117.22	10	15	III	4169.90	6	10	III
118.426†	8	12	III	4170.46	10	25	III A
119.987†	20	30	II	4171.04	20	5	III
120.43	3	6	III A	4171.78	12	15	III
	15		VE	4172.55	15	2	III
120.51		12	III A, 2		10	50	III A
121.04	5 8	1	III A, 2	4172.83		2	III
122.49		10		4173.44	10		III A
123.80	5		V E, 2	4175.05	4	15	
124.25	2	3	III	4175.86	8	15	III A
125.25	15		V E, 6	4178.97	15		VE, 4
125.38	4	2	III	4180.40	10	40	III A
126.72	10	8	III	4180.88	15	2	III, 3 III A
.126.93	4	3	III	4184.66	2	4	III A
	54	4	III	4185.44	2	1	III
127.29	10		VE	4185.89	5	15	III A
128.72	3	2	III	4186.24	15	10	III
120.40	3	ī	III	4187.16	25	60	II A, 2
	20	25	III	4188.12	15	40	III A
130.15	8		III	4188.53	8	40	VE
131.11	6	4	III A	4180.30		7.5	III A
132.51	8	15			3	15	III
132.84	0	8	III	4190.43	2	3	TIL

# ARTHUR S. KING

# TABLE III—Continued

	Inti	ENSITIES	CLASS,		INT	ENSITIES	CLASS,
λ (I.A.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comp
4191.06	20	15	II	4266.34	50	80	II
4193.34	5	10	III A	4269.34	6	30	III A, 2
4104.02	5	5	III	4269.72	25	40	III
4196.74	15	25	III	4270.74	3	2	III
4198.45	5	15	III A	4273.19	3		VE
4198.99†	5	10	III A	4275.23†	20	20	III
4199.07†	3	10	III A	4275-391	5	8	III
	3	10	III	4276.14	2		VE
4201.00*	30	2	VE	4276.75	15		V E, 6
4203.74	40	60	II	4277.78†	6	8	III
4206.49	25	20	II	4278.54	200		VE, 6
4207.54	8	20	III A	4281.31	4	6	III
1208.70	5	5	III	4285.12	25		VE
1211.13	3	5	III	4285.74	5		VE
1213.50	∫20	50	III A	4286.89	8		V E, 2
1213.50	130		V E, 4	4287.10	8	15	III A
214.42	15		VE	4287.31	4		VE
216.69	2	8	III A	4289.70	20	40	III A
1218.85	8	15	III A	4201.30	2	4	III A
210.10	20	30	III A	4292.63	2		VE
220.12	6	3-	V E, 2	4293.14	6	8	III. 2
222.71	15	10	III	4294-34	5		V E, 2
223.34	12	-	VE	4295.34	15	30	III A
224.29	25	30	II	4296.34	10	4	III A
224.85	4	30	V E, 2	4298.36	30	60	II A
.226.45	60		V E, 6r	4290.89	15	15	III
230.60		6	III, 2		12	10	III
0	4		III, 2	4302.95		8	III
231.34	5	20	III	4303.99	10		III, 2
231.89	20	40		4304.24	4	80?	
232.20	20		VE	4307.19*	15	801	II A
232.84	25	20	III	4308.68	25		VE, 6
235-35	30	60	III A, 2	4310.44	20	30	III
239.29	15	40	III A	4310.65*	3	3	III
239.91	. 8	30	III A	4310.99	8		VE
240.14	15		V E, 6v	4311.29	5	10	III A
242.27	8	6	III	4311.58	15	20	III
242.57	12		VE	4312.09	8		VE
245.19	10	20	III A	4313.25	15	20	III
246.60	20	60	III A	4313.41	10	12	III
248.59	4		VE	4315.74	6	8	III
251.34	15	15	III	4318.84	200	200	I
252.66	5	12	III A	4320.27	12	30	III A
254.02	4	12	III A	4321.50	4	3	III
255.24	25	80	III A, 2	4322.23	50	50	III
256.10	8	20	III A	4322.87	15	15	III
258.24	30	60	III A	4323.65	15		VE, 4
259.94	5	20	III A	4324.09	8	20	III A
261.84	6	10	III	4325.50	10	60	III A
263.56†	6	15	III A	4325.82	60	4	IVE, 4
263.601	15	25	III A	4326.48	200	200	I I, 4
	6	25	III A	4328.08		200	VE
264.73†	6		III A, 2	10	I	20	III A
264.98	0	20	111 /1, 2	4328.94	10	20	TILLY

TABLE III—Continued

	INTE	ENSITIES	CLASS,		Inti	ENSITIES	CLASS,
λ (Ι.Α.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comp
4329.40	4	5	III	4384.06	5		VE
4329.56	8	5	III	4385.60	12	10	III
4330.34	8	4	III, 2	4386.09	10	1	VE
4330.83	2	10	III A	4387.43	2	15	III A
4330.94	3	12	III A	4388.25			III
			III		15	20	
4332.13	50	50		4389.04	3		V E, 2
4333-73	8	20	III A	4389.94	4	2	III
4334.69	12	20	III	4390.93*	25	30	III, 2
4335.74	2		VE	4392.23	8	8	III
4335.89	6	8	II, 2	4392.96	3	2	III, 2
4336.52	60	80	II	4394.04	5	15	III A
4337.64	40	50	III	4394-92	3		VE
1338.47	150	150	I	4396.58	10	8	III
1339.61	8	15	III A, 2	4398.61	2	4	III A
1339.88	4		III	4400.52*	2	2?	III ?
1340.63	100	150	I			2.	
				4401.54	10		VE, 2
1340.99	10	15	II	4403.20	10	10	III
1342.5*†	30		VE, 6	4405.41	8		V E, 4
342.526*†	83	8	III	4409.51	20		V E, 6v
342.595*1	20	20	III	4411.93	2	3	III, 2
343.29	1	2	III A	4412.83	2	I	III
343.85	3	3	III, 2	4413.63	4	6	III
344.20	3		V E, 2	4416.28	20		V E, 4
348.34	3		VE	4420.19	15	100	III A
349.60	8	8	III	4423.11	40	200	IIA
350.74	6	10	III		,		III A
351.57	8	10		4427.391†	4	12	
		******	V E, 4	4428.33	2	4	III A ·
353.19	50		V E, 4	4430.13	2		VE
356.08	20	20	III	4430.740	3	6	III A
356.83	100	100	I	4432.164†	3	6	III A
357.48	10	30	III A	4432.722†	12	40	II A
358.42	3	2	III	4434.48	10		V E, 2
360.17	20	30	III	4436.13*	20	100 ?	III A
360.66	3	6	III A	4437.65	2	2	III
362.47	5	10	III A	4438.98	10		VE
363.94	I	I	III	4439-39†	8	10	III
366.01	3		VE	4441.27	6	10	V E, 4
367.32	25	2	III	4441.49		8	III
370.33	4	I	III		5		
			VE	4448.04	30	20	III, 4
370.93	3			4449-23	3	10	III A
372.07	20	50	II A	4449-51	8	4	IV, 4
372-34	2	Ι,	III	4450.11	2		VE
372.50	4		VE	4451.63	12	15	III
372.63	2		VE	4452.82	30	15	III, 6
374.46	4	10	III A	4458.45	2		VÉ
374.83	2	10	III A, 2	4459.38	8		VE
375.36	3	2	III	4461.23	8	4	III, 4
375.62	3		VE	4462.18	10	4	V E, 4
376.44	5		V E, 2			6	
378.73			VE.	4465.69	2		III A
	8		VE, 4	4467.69	10	50	III A
381.32		25	III A	4469.13	8	30	III A
382.45	20	40	III A, 2	4469.68	8		VE, 4

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TABLE III-Continued

) (TA)	INTE	ENSITIES	CLASS,		INTE	ENSITIES	CLASS,
λ (Ι.Α.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Com
4471.72	5		VE	4541.24	I	4	III A
1472.84	3		VE	4542.4*†	4		VE, 4
473.67	4	20	III A	4544-233†	I	3	IVA
476.429†	2	20	III A	4547.682†	1	6	III A
480.04	2		VE	4549.07	10	6	IV. 4
481.03	10		VE	4549.138*†	5	10	III A
485.67		10	III A	4549.70	8	10	V E, 4
	4	100	III A. 2	4550.450†	12	40	III A,
488.16	12		VE	100 .0 .	20	40	III, 4
489.76	3			4556.42		4	VE.
490.63	8		VE, 4	4556.92	8		VE, 4
491.02	10	20	III A	4556.962*†	5	20	
491.78	2	2	III, 4	4557.230†	4	10	III A
493.07	100	200	II A	4557-29	2	******	VE
495.03	2		V E, 2	4557.635†	6	10	III
496.93	1	1	III	4560.7751	3	20	III A
498.96	1		VE	4562.25	8		V E, 2
499.47	2	I	III	4563.68	20		V E, 2
501.26	2		VE	4564.83	4		V E, 2
501.73	3	15	III A	4567.72	6		VE, 6
503.58	2		VE	4568.523†	6	8	III
504.59	3	8	III A	4569.29	2		VE, 2
507.97	2	4	III A	4571.42	1		VE
508.67	I	2	III A	4571.83	2	2	III
509.03	15	-	V E, 4	4573.18	8		VE
511.53	20	20	III, 4	4573.895†	4	8	III A
0 00	8	20	V E, 2	4575.42	2		VE
512.97		8	III A				V E, 4
513.639†	I		VE	4578.62	30	40	III A
514.308†	8			4581.42	6	40	VE
515.87	6		VE, 4	4582.57			VE
516.145	I	3	III A	4584.82	6		
516.532†	1	12	III A	4587.7101	6	8	III, 4
518.214†	2	I	III	4591.56	8	******	VE
520.09	4		VE	4592.41	4	3	III, 2
521.839†	2	3	III	4593-943	2	10	III A
524.133†	3	I	IV	4599.607†	2	15	III A,
524-333†	4	15	III A	4600.162	6	15	III A
524.99	8		V E, 4	4604.08	3		VE
525.946†	2	6	III A	4611.92	2	5	III A,
26.449†	1	3	III A	4617.27	3	2	III
527.18o†	1	5	III A	4620.88	2	I	III
28.323	I	4	III A	4621.97	1	3	III A
530.658†	2	8	III A, 2	4626.32	6		V E, 4
531.83	6		V E, 4	4626.91	15		VE
32.920†	2	8	III A	4627.52	2	I	III
		I	III	4631.038†	2	8	III A
534.13	5		III A		8	10	III, 4
34.290	8	3		4632.02	8	10	III, 4 III, 4
36.93			VE, 4	4636.61		10	VE
537.15	8	20	III A	4637.00	3	0	VE, 2
537.23	8	10	III	4638.516†	4	8	III A
38.74	2		VE	4640.98	15		VE
540.23	4		VE	4641.97	30		V E, 2
40.58	3	1	VE, 4	4645.26*	50	1	VE, 6

TABLE III—Continued

> (T.A.)	Intensities		CLASS.		Intensities		CLASS,
λ (1.Α.)	λ (I.A.)	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comps
4648.30	2 I	1	III V E	4658.67 4662.78	3	4 30	III A
4651.58 4658.35	2	5	III A III A, 4	4676.86	10	40	III A, 4

	NOTES TO TABLE III
λ	
3847.88]	
to }	Starred lines uncertain on account of $\lambda$ 3883 CN band
3881.75	
3913.79	Blend $Tb$ I (to violet) and $Tb$ II
4013.276	Blend complex Tb II line
4201.00	Close blend $Tb$ I and $Tb$ II
4307.19	Furnace line partly V
4310.65	Blend $Tb$ II in arc
4342.5	
4342.526	Difficult blend
4342.595	
4390.93	Blend in arc with $Gd$ 11
4400.52	Blend $V$ in furnace
4436.13	Blend $V$ and $Gd$ I
4542.4	Components partially resolved
4549.138	Blend in arc with \(\lambda\) 4549.07
4556.962	Blend in arc with $\lambda$ 4556.92
4645.26	Components partially resolved

TABLE IV
TEMPERATURE CLASSIFICATION OF DYSPROSIUM LINES

	INT	ENSITIES	Crase		INT	ENSITIES	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
3836.49	60	5?	III E	3903-332†	3	4	III
3838.67	5		VE	3904.089	I	4	III A
3840.01		5?	II	3004.14	8		VE
3841.313†	40	3	IVE	3005.56	2	1	VE
842.015†	20	2?	III	3910.108†	I		III A
			II			5	VE
3844.30	6	3		3910.51	I		
846.351†	15		VE	3911.677†	I	4	III A
846.99	8	6	II	3912.544	2	4	III A
848.36	2	3	III	3912.85	3		VE
849.399	15		VE	3913.628†	4	8	III A
850.0491	5		VE	3913.95	6		VE
850.449†	4		VE	3014.8691	25	I	VE
850.527†	5		VE	3915.599†	25	I	VE
853.040	50	3	IVE	3917.294	8	8	III
			VE			6	III
854.90	4			3917-372†	6	0	
855.60	5		VE	3918.53	4		VE
858.08	5	*****	VE	3919.145	2	6	III A
858.39	15	8?	II	3920.1221	1	3	III A
862.66	5	3	III	3920.84	I	3	III A
863.20	4		VE	3023.3001	3		VE
865.45	8		VE	3923.390†	10		VE
866.590	20		VE	3924.415†	1	3	III A
867.83	5		VE	3924.46	2	3	VE
868.464†	25	2?	IVE	3927.866†		4	III
	-	12?	II		4		VE
868.817†	50			3929.33	5		
869.10	3		VE	3930.153	15	10	III
869.439†	10		VE	3931.297†	8		VE
869.871†	25	3	VE	3931.533	150	5	IV E
871.642†	10		VE	3932.231	10		VE
872.14	300	3	IVE	3932-97	5		VE
873.989†	40	3	IVE	3934.17	8		VE
875.15	4		VE	3936.03	4		VE
877.94	2		VE	3936.29	4		VE
879.05*	20		VE	3936.708†	10	8	III
			VE				III
881.995†	12			3937.165	4	4	
883.05	4		VE	3937-9901	3	5	III
887.52	8	I	IV	3938.17	4		VE
388.40	6		VE.	3938.209	5	5	III
388.99	12	2	IV	3939.23	I	3	IV A
891.85	3		VE	3939-793†	1	5 -	IV A
891.9821	6	5	III	3942.050†	4		VE
892.87*	12	8?	II ?	3942.527†	25	1	VE
393.390†	I	6	III A	3944.688†	600	15	HE
		6	III A				VE
94.533†	2			3946.939†	10		
95.35	15		VE	3950-395	20		VE
396.18	1	6	III A	3953-515	I	6	III A
396.656†	2	6	III A	3954-559	12		VE
398.540	300	20	IVE	3957-797†	30	I	VE
		54	III	3958.004	2	4	III A
399.15	6	16	III	3959-35	3		VE
02.39	2	10	VE	3961.810†	1	5	III A

TABLE IV—Continued

	Int	ENSITIES			INT	ENSITIES	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CL
3962.601†	8	8	III	4020.88	4		VE
3963.80	3		VE	4020.897†		4	III
3964.70	2		VE	4023.7221		15	III
3966.362†	3	6	III A	4024.437			VF
3967.517†	10	8	III			6	
	-			4024.906†		6	III
1968.393†	1000	20	HIE	4025.205		4	III
969.233†	3	5	III	4025.605*†	3	4	III
971.214*†	3	6	III A	4025.75	3		VE
972.414†	2	5	III A	4027.7871			VE
973.8791	5	6	III	4028.325†			VE
976.965†	I	4	III A	4028.418†	12	8	II
978.556†			IVE	4020.410			-
	150	5		4031.081	2	4	III
979-477	8	******	VE	4032.480	30		VE
981.372†	3	6	III A	4032.847†	4	4	III
981.930†	75	2	VE	4033.588†	I	4	III
983.6591	100	5	IV E	4033.666†	10		VE
984.221	60	I	VE	4036.335†	15	I	VE
984.70	2		VE	4037.628†			
		1	III A		I	4	III
985.360†	1	4		4038.528†	8		VE
986.05	2	4	III A	4038.719†	I	4	IIIA
987.06	3		VE	4038.834†	4	6	III
988.886†	2	5	III A	4041.989	10		VE
990.35	2		VE	4043.046†	1	3	III A
991.328†	30		VE	4045.281†	2		III A
991.89	2		VE			5 p	
				4045.977	400	300R	II
993.575	4	4	III	4047.740	2	4	III A
993.857	1	3	III A	4048.942	4	4	III
994-5351	1	6	III A	4049-3741	3	3	II
995-993†	1	6	III A	4050.581†	150	3	IV E
996.694†	200	4	IVE	4052.40	I	3	III A
006.026†	I	4	III A	4055.013†			III
008.10	2	4	VE		3	5	-
			III A	4055.13	30	******	VE
998.940	I	5		4057.40	2	******	VE
000.454	600	10	III E	4060.58	4		VE
000.588†	1	3	III A	4061.054†	I	3	III A
01.524	1	3	III A	4065.15	I	2	III A
04.48	2	3	III	4065.400†	2	5	III A
o5.840†	12	6	III	4066.70	I	4	III A
06.071	10	6	III	4067.968†			III A
		_			1	4	
07.137†	2	4	III A	4071.032†	2	4	III A
07.77	3		VE ·	4072.65	3		VE
10.08	3		VE	4073.110	200	10	IV E
11.295†	20		VE	4074.02	3		VE
11.8201	1	3	III A	4077-35	I	3	IVA
12.523†	ī		III A		600		III E
		4		4077.974†		20	
13.826†	15	40r	II A	4079.27	2	5	III A
14.094	I	4	III A	4079.595†	2	5 "	III A
14.713†	25	1	VE	4083.109	2	6	III A
15.18	4		VE	4083.74	1	4	III A
17.055†	I	4	IIIA	4085.140†	6	10	III A
19.40	ī	-	VE	4085.344†	12	8	Ш
						0	
19.55	1		VE	4087.22	8		VE

# ARTHUR S. KING

TABLE IV-Continued

) ([A)	INTE	NSITIES			INTE	NSITIES	
λ (I.A.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
1087.388†	2	6	III A	4154.22	2		VE
089.511†	I	5	III A	4154.27†	1-	4	III A
090.399†	1-	3	III A	4154-54	1-	4	III A
091.53	6		VE	4156.39	1-	3	III A
091.77	4	1	VE	4156.98	I	6	III A
093.647†	I	5	III A	4157.86	I		VE
	1-		III A				III A
095.251†	8	3	III A	4158.04	I	5	
096.109†		15		4159.34	3	6	III A
096.639†	1	5	III A	4160.24	I	3	III A
099.886†	2	4	II A	4162.25	I	4	III A
101.43	1	4	III A	4164.74	1-	4	III A
101.850	1-	3	III A	4166.23	I	3	III A
101.95	2		VE	4167.40	2		VE
103.3127	500	15	IVE	4167.966†	200	100R	II
103.878*†	30	6	III	4169.07	1	5	III A
105.032*†	3	5	III A	4169.241†	2	3	VE
105.05	8	3	VE	4170.34	1-	5	III A
105.814†	I	2	III A		I	1	IVA
		3	VE	4170.55		8	III A
106.39	3			4171.925	4		
106.70	2		VE	4171.992†	6	8	III
107.45	2		VE	4176.64	I	3	III A
111.346†	150	8	IV E	4176.81	1	3	III A
113.05	2	8	III A	4177-754†	2	3	III
114.09	1		VE	4178.072†	5	4	III
114.69	I	6	IV A	4181.25	2	5	III A
119.33	8		VE	4182.42	2		VE
120.66	1	4	III A	4183.611†	6	10?	III
124.626†	30	I	VE	4183.726†	30	30r	II
126.14	4	157	IIA	4186.8101	600	300R	II
128.241†	20	1	IVE	4189	2		III A
	8	1	III A	4109		8	III
129.13		15r	IVE	4190.90	10		III
129.425†	100	3		4191.627	80	50R	
30.42	6	127	II A	4194.827†	500	200R	II
131.04	6		VE	4195.22*	6	3	III
132.85	3		VE	4197.93	25	15	II
133.38	8		VE	4201.01	2	5	III A
33.86	10	6	III	4201.314†	20	15	III
34.14	8	4	III	4201.372†	10		VE
34.75	2	3	III	4202.245	30	201	II
36.94	1	ĭ	IV	4203.99	I	4	III A
38.54	3	6	III A	4205.03	8	10	III
39.56	3	6	III A	4206.544*†	25	6	IVE
41.395†	1	6	III A	4207.68	5	6	III
	20	2	IVE	4211.20		307	II A
41.514†			IVE		15		II
43.100†	300	5			1200	600R	
45.59	I		VE	4213.182†	60	50R	II
46.069	40	30r	II	4214.38	4	12	III A
48.00	1	4	III	4215.166†	125	100R	II
49.84	I	4	III	4216.96	1	4	III A
52.43	1		VE	4218.001	200	150R	II
53.11	3	4	III	4218.58	5		VE
						200R	

TABLE IV-Continued

	INTE	NSITIES			INTE	NSITIES	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
222.21	20	15	III	4277.76†	1-	3	III A
224.68	2	4	III A	4279.78†	2	8	III A
1225.153†	150	100R	II	4282.11†	1-	3	III A
1232.02	30	15	II	4283.31†	1-		III A
1234.14	1		III A	4283.63†	1-	5	III A
		4	III A			5	
234.83	2	4		4284.28†	1-	3	III A
236.27	1	4	III A	4288.01 †	1-	3	III A
236.59†	1-	4	III A	4288.3†	1-	2	III A
237.54	2		VE	4288.71†	1-	3	III A
237.95	1-	3	III A	4290.44	2	8	III A
238.04	I		VE	4291.01	1-	3	III A
238.44	2		VE	4291-93*†	8	15	III A
239.87†	40	30r	II	4294.45†	1-	2	III A
240.69	1-	2	III A	4294.936†	10		VE
241.83	i-	5	III A	4295.038†	25		III
242.73	1-		III A			7	III A
		4		4296.34†	1-	3	
243.00	1	4	III A	4297.76†	1-	3	III A
243.45	2	10	III A	4298.7†	1-	3	III A
244-79	1		VE	4298.90†	I	5	III A
245.92	30	20	III	4300.41	I		VE
247.36	8		VE	4300.6 †	1-	4	III A
248.44	2		VE	4300.76	1		VE
248.91	1-	2	III A	4302.35†	1-	2	III A
250.35	I	3	III A	4302.57	4		VE
250.46†	1	5	III A	4302.72	2		VE
251.33†	I	8	III A	4306.77†	1-	3	III
251.73†	2	6	III A	4308.344†	2		III A
252.11	1-		III A		100	5	IVE
		3	III A	4308.623†		7	
254.927	1 —	3		4311.93†	1	4	III A
255.97	1		VE	4312.429	1-	4	III A
250.204	3		VE	4313.886†	2		VE
256.323†	15	3	IV E	4313.929†	1-	6	III A
257.72	I		VE	4315.60†	1-	4	III A
257.78†	1 —	10	III A	4318.355	1-	4	III A
258.14	4	10	III A	4318.985†	1-	4	III A
258.51 †	1-	5	III A	4322.10	1		VE
258.56†	8	10	III	4322.363†	1	4	III A
259.76†	1-	3	III A	4322.55	2		VE
259.82†	1	3	III A	4323.794*†	ī	5	III A
260.71	1-	2	III A	4325.14	10		VE
264.82	1-	2	III A	4326.30	2	4	III A
265.83			VE				III A
	I			4328.69†	ı —	2	
266.00 †	1-	4	III A	4328.90	5		VE
267.92†	1	3	IVA	4329.89	1		VE
268.31 †	2	10	III A	4334.07	1-	3	III A
269.60†	3	8	III A	4334-37†	1-	6	III A
273.03 †	1-	4	III A	4336.01	1-	6	III A
273.14	2		VE	4339.68	8		VE
275.00	I	6	III A	4346.3	I		VE
275-45† · · · ·	I	3	III A	4347-721	4	8	III A
275.96†	1-	3	III A	4356.13†	i-	6	III A
276.74	10	15	III A		1-		III A
1 1	10	-3	*** **	4350.771	4	3	TIT W

TABLE IV-Continued

	INTE	NSITIES			INTE	NSITIES	
λ (Ι.Α.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc	Fur.	CLASS
358.47†	12	2	IV E	4506.96	1	2	IV A
361.39	2		VE	4508.11	1-	3	IV A
364.06	2		VE	4513.60	1-	12	III A
364.28	5		VE	4516.94*	3	5	III
365.62†	1-	4	III A	4518.53	3		VE
366.73	5	12	III A	4519.79	2		VE
			III A		1-		III A
368.26†	1-	2	III A	4522.68		4	
369.55	r —	6		4525.10	1	3	IV A
374-24	20	3	IVE	4526.09	I	3	IV A
374.80	20	2	IVE	4527.74	6		VE
375-33	6		VE -	4531.59	I	8	IIA
380.23	2		VE	4538.74	5		VE
380.48†	1-	5	III A	4539.14	1-	6	IV A
384.30	2		VE	4541.69	4		VE
385.29	2		VE	4545.35	I		VE
389.79†	I	3	IV A	4546.48	1-	2	IV A
390.32	1-	2	III A	4546.82	1-	3	III A
390.94	1-	5	III A	4550.90	2	3	VE
393.28†	1-		III A		1-	8	III A
		3	VE	4553.15			III A
394.98	6			4555.02	1 —	4	
397.51	1-	4	III A	4555.23	I	8	III A
399.73	1 —	4	III A	4556.48	1		VE
400.10	1		VE	4557.36	1-	4	III A
403.98†	1-	4	III A	4559.62	I -	3	III A
408.05	2		VE	4565.098†	8	20r	II A
409.384†	40	5	IVE	4565.95†	1-	5	III A
400.03†	1-	2	III A	4566.22	1-	3 .	IV A
411.37	I	3	IV A	4567.06	1	8	III A
411.69†	1-	3	III A	4568.42	1-	3	III A
414.20	i —	3	III A	4573.83	2		VE
			VE				VE
421.69	I		IVA		2	D	II
426.87	1	3		4577-7771	30	40R	
431.00	3		VE	4585.73	1-	5	III A
435.78	1	* * * * * * *	VE	4585.99	1-	4	III A
436.65	I		VE	4586.18	1 -	3	III A
444.61	6	12	III A	4586.64	1	I	IV
448.23	2		VE	4587.93	5		VE
449.16	2		VE	4589.078†	I	12	III A
449.702†	60	7	IVE	4589.3671	150	150R	II
454-334† · · ·	1-	4	III A	4590.55	I	3	III A
455-49	4		VE	4591.6†	1-	3	III A
467.88	2		VE	4591.73	1		VE
468.16*	8		IVE	4595.13	r		VE
	2	3	III A		i-	6	IVA
480.68	-	10		4599.84			III A
482.33	1	2	IVA	4606.04	1	4	
484.37	2	10	III A	4612.27	100	80	II
487.85	1-	2	III A	4613.83	1	5	IV A
492.14	1-	5	III A	4614.82	1	6	IV A
196.41	I	2	IV A	4617.25	4		VE
199.26	I	2	III A	4620.02	10		VE
503.26	5		VE	4622.34	1		VE

TABLE IV-Continued

λ (I.A.)	Intensities			200	Inte	CLASS	
A (L.A.)	Arc	Fur.	CLASS	λ (Ι.Α.)	Arc		CLASS
4624.10	1	5	IV A	4650.17	I	12	III A
4627.45	1-	5 8	III A	4651.54	2		VE
4628.07	1		VE	4662.20	I	3	III A
4631.51	1	8	III A	4662.76	I	4	III A
4634.78	1-	3	IV A	4664.68	5	1	IV E
4635.32	1	6	III A	4673.62	2		VE
1638.52	I		VE	4675.81	2	2	IV
641.1†	1-	4	III A	4681.01	1	5	III A
4643.49	1	4	III A				

#### NOTES TO TABLE IV

λ	
3879.05	Double

<sup>4206.544</sup> Probably blend with Dy I

<sup>4291.93</sup> Double

<sup>4323.794</sup> Blend in arc with faint Dy II

<sup>4516.94</sup> Blend in arc with Dy II

 $\begin{tabular}{ll} TABLE & V \\ TEMPERATURE & CLASSIFICATION OF HOLMIUM & LINES \\ \end{tabular}$ 

	INTE	NSITIES	CLASS,		INTE	CLASS,	
λ (Ι.Α.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comps
3837.45	15		V E, 6	3975.88	3	4	III
3842.05	4		V E, 4	3976.86	6	6	III, 2
3843.86	8		V E, 2	3977.05	3		VE, 4
846.68	10		VE, 4	3985.80	6		VE, 6
851.47	4		VE, 4	3986.5*†	5		VE
854.08	20		V E, 6	3987.55	3		VE?,
856.98	8		VE, 6	3990.35	1		VE
861.7*†	40	3	V E, 6	3991.85	1		VE
874.11	2		V E, 4	3992.55	4	6	III, 6.
874.70	6		V E, 4	3998.28	40	20	II, 4
886.4†	6		V E, 6	3999.56	6	8	III, 2
888.95	40	8	IVE, 6	4000.62	4?	2	III
890.42	5	8	III	4002.70	3		VE, 6
891.02	200	40	IV E, 6v	4003.36	4	6	III, 2
892.70	2	6	III A	4014.18	2		VE, 2
892.95	6	12	III A	4016.00	2	4	III A, 2
893.10	4		VE, 6	4020.51	1	2	III A
893.54	5		VE, 6	4022.86	2		V E, 6r
895.53	4		VE	4023.93	4		V E, 6r
896.24	4	2	III	4025.39	3	4	III
896.6†	5	3	III	4027.20	5	6	III, 2
896.75	5		V E, 4	4028.85	4	3	III, 2
897.27	4		V E, 6	4031.75	4	15	II A, 4
900.79	I	2	III A, 2	4036.10	2	4	III A, 4
900.79	I	I	III, 2	4040.84*	150	100R	I, 2
901.53	I	I	III	4045-43	200	40	IVE, 4
902.24	4		VE?, 6	4048.79	2	7-	VE, 4
		4	III	4052.45	1	I	III
3904.45	4	4	IVE, 6	4053.92*	400	300R	I. 2
3905.76	30	4	VE.	4054.49	3	3	IV
909.56	3	10	III A, 4	4057-55	2	3	III, 2
910.30	1	10	III	4061.02*	I	3	III A
011.80		6	III A, 2	4065.10	10	I	IV E
	3	2	III	4067.04	2		V E?, 2
912.44	4		VE	4067.8†	2		V E?, 6
913.96	3	6	III	4068.05	6	10	III, 2
919.45		0	VE	4087.35	4	5	III
923.37	5		VE, 6	4087.65*	4	6	II, 2
936.51	10	· · · · · ·	III A	4100.27	4	4	III, 4
937.00	2	6	III A, 4	4100.59	I	4	VE
938.84	2			4101.09	40	30	I, 2
940.55	12		V E, 6v		400	300R	I, 2
949.00	I	I		4103.84*	2	2	III
950.50*	83	6	III, 2	4104.30			III A
955.05	4	I	III, 2	4105.85	2	4	III
955-74	15	20	I, 2	4106.50	4	4	V E, 4
957-39	I	3	III A, 2	4106.6†	2		III A
959.68	6	8	III, 2	4107.36	I	3	II
963.27	1	I	III	4108.63	100	6or	
972.60	2		V, 4	4111.99	4	6	III
973.05	I	4	III A	4116.63	3	3	III
973.59	1	4	III A	4120.20	50	30	II

TABLE V-Continued

λ (Ι.Α.)	INTE	ENSITIES	CLASS,	CLASS.		ENSITIES	CLASS.
A (1.73.)	Arc	Fur.	No. Comps.	λ (Ι.Α.)	Arc	Fur.	No. Comps
4125.65	20	30	'III'	4348.34	I	I	III
4127.16*	150	60	II, 2	4350.73	40	40	I
4134.56*	5	5?	III?, 2	4363.93*	2	3	IV ?, 4
4136.24	40	30	II	4364.18	3		VE
4142.19	2	3	III, 2	4369.17	1	4	III A, 2
4148.97	3	3	III, 2	4373-33	I	3	III A
4150.8†	2	6	III A, 6	4384.94	3		V, 6
4151.14	4	6	II	4390.53	2	4	III A, 2
4152.54*	30	56	IV, 6r	4400.54*	1	3	?, 2
4152.75*	30	3	IV, 6v	4403.27	3	8	III A, 2
4160.0†	3	6	IV A, 4	4420.54	2		VE
4163.03	100	8or	I	4455.59*	5		V, 4
1172.23	2	4	III A, 2	4477.63	3	I	III
4173.23*	50	40r	I	4497.7†	2	25	III A, 6
1194-34	30	60	I, 6	4510.81	2	2	III, 4
1223.47	4	5	III	4512.54	I		V
1229-49	6	2	IV, 2	4526.14	2	1	III, 2
1234.79	2	2	III	4531.28	2	2	III, 4
1243.76	3	6	III A	4534.58	4	12	III A, 4
254.43	100	100	I, 6	4562.52	6	15	III A, 6
1264.07	15	40r	IA	4572.43	2		V, 4
266.04	5	20	III A, 2	4606.62*	I	5	III A
1273.63	1		V, 2	4608.0 †	2		V, 4
1284.54	1	3	III A, 2	4609.52	6		V, 6
1290.17	1		VE	4613.37	4	12	III A, 4
301.09	2		V, 4	4618.84	2	2	III, 4
311.04	2	4	III A, 2	4628.22	2	10	III A, 4
326.39	2	2	III	4629.10	6		VE
328.99	3	******	V	4640.61	2		V
1330.64	2	3	IV ?, 4	4641.7†	1	12	III A, 6
1337.13	4		VE	4661.33*	4	3	III

#### NOTES TO TABLE V

- 3861.7 Blend head of CN band
- 3950.50 Blend V in arc
- 3986.5 Probably two complex lines together
- 4040.84 Reversal is for red component, which is complex
- 4053.92 May have four close components
- 4061.02 May be impurity
- 4087.65 Red component shows at low temperature
- 4103.84 Resembles λ 4053.92
- 4127.16 Red component stronger in furnace
- 4134.56 Blend V in furnace
- 4152.54\ Apparently a blend of two six-component lines, with components oppositely
- 4152.75) graduated
- 4173.23 Blends in furnace with foreign line
- 4363.93 Disturbed by band
- 4400.54 Blend V in furnace
- 4455.59 Given by Kayser as welsium
- 4606.62 May be impurity
- 4661.33 Disturbed by band

#### DISCUSSION

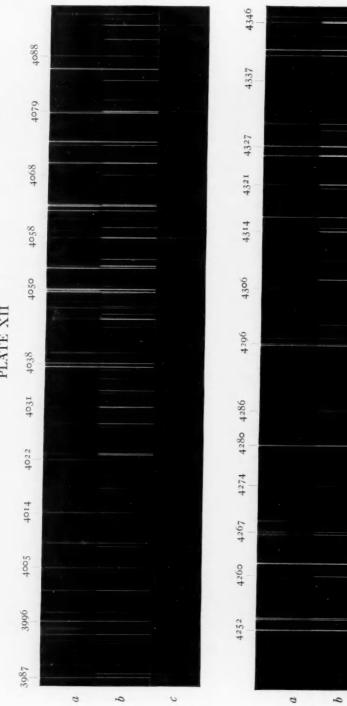
Europium.—Table I contains 219 lines, of which 133 belong to the neutral and 86 to the ionized atom. Each of these main divisions contains two groups of very different type, one group responding to low and the other only to high excitation, with a lack, in this portion of the spectrum, of lines of intermediate character. Hyperfine structure prevails for a large proportion of the lines, the patterns being narrow and seldom sufficiently resolved to permit more than a rough estimate, from the appearance of the line, of the probable number of components. That hyperfine structure, rather than the widening due to high excitation, is responsible for the soft and diffuse appearance of many lines in the arc spectrum is indicated by the failure of these lines to sharpen when barely visible in the furnace spectrum, where excitation widening should disappear.

The lines of low excitation in the ionized spectrum have intensities seldom found in rare-earth spectra. These are  $\lambda\lambda$  3907.1, 3930.5, 3971.9, 4129.7, 4205.0, 4435.6, 4522.6. The line  $\lambda$  3819.6, outside the range here studied, belongs to the group. In structure each line consists of an unresolved triplet, with a fourth line, usually separated in the furnace spectrum, on the violet side. The complex line reverses strongly in the arc, the single line to the violet making this side of the reversal stronger. The ionized character of these lines was clearly shown by their almost complete extinction when caesium was added in the furnace; but with europium alone the lines are strong in the furnace, and with long exposure show at temperatures as low as 2000° C. This behavior places them in class III E. The other enhanced lines in this region are of very different type. These were selected by comparison of arc and spark spectra and require high excitation, belonging usually to class V E.

The neutral lines of europium present a similar contrast between groups. Three low-temperature lines occur at  $\lambda\lambda$  4593.993, 4627.122, 4661.865. These reverse widely in the arc and at the higher furnace temperatures. At low temperatures they widen strongly while the vapor is plentiful, after the manner of "flame" lines, but the strong sharp line which persists at low vapor density in each case indicates an absence of hyperfine structure. Aside from these three, the neutral lines in this region are weak in the furnace, and appear faint-



PLATE XII



SPECTRUM OF GADOLINIUM

2

- a) Spark spectrum
- b) Arc spectrum
- c) Furnace spectrum

ly at high temperature, though some of the strongest can be seen at temperatures as low as 2300° C.

The wave-lengths in Table I are for the most part by Eder. The lines belonging to the strong groups discussed above have been measured by the writer in the furnace spectrum, where superior definition of the components of the complex ionized lines was obtained in spectrograms taken at moderate temperature. Standards were furnished by lines of titanium and vanadium given by the graphite tube.

Gadolinium.—Table II contains 627 lines, of which 365 are neutral, while 262 belong to the ionized atom. The wave-lengths are for the most part the international values of Eder,² but are supplemented by 236 wave-lengths measured on the writer's spectrograms. The spectrum is typical of the even-numbered rare earths, its lines being sharp and reversing with difficulty. The lines of class I are visible at 2000° C, and the low-temperature spectrum is well developed at 2100°. Between this and the spectrum at 2300° there is a decided contrast, owing to the development of class III lines. The stronger ionized lines show at 2600°, but were fully suppressed at this temperature by the addition of caesium chloride. Spectra of the spark, arc, and high-temperature furnace are reproduced in Plate XII, which illustrates the slight difference between the arc and spark spectra in this region, other than the usual weakening in the spark of the neutral lines forming the furnace spectrum.

Terbium.—The region under examination is very rich in lines, 733 being of sufficient strength to be listed in Table III. The wavelengths are for the most part converted from those of Exner and Haschek,<sup>3</sup> but some are from Eder<sup>4</sup> and from Eberhard,<sup>5</sup> and 128 lines, a part of them new, have been measured by the writer. The numbers of neutral and ionized lines are 482 and 251, respectively. The furnace spectrograms showed a small number of class I lines, of which several are grouped near  $\lambda$  4300; but the very rapid development at medium temperature places a large proportion of the furnace lines in class III.

<sup>&</sup>lt;sup>1</sup> Sitzungsberichte der Akademie der Wissenschaften in Wien, IIa, 126, 473, 1917.

<sup>&</sup>lt;sup>2</sup> Ibid., 125, 1467, 1916. <sup>3</sup> Loc. cit. <sup>4</sup> Op. cit., 131, 199, 1922.

<sup>5</sup> Zeitschrift für wissenschaftliche Photographie, 4, 137, 1906.

Hyperfine structure, while not a conspicuous feature of this spectrum, is present in a considerable number of lines (see final column of Table III). The patterns are in general narrow. The lines given as having two components may be more complex, and occasional patterns of six components, with partial resolutions, are found.

Dysprosium.—The wave-lengths in Table IV, which contains 534 lines, are for the most part those of Eder, who also lists the converted values of Exner and Haschek<sup>2</sup> and of Eberhard.<sup>3</sup> These are supplemented by 274 wave-lengths obtained from the writer's plates, chiefly for lines which can be measured to advantage in the furnace spectrum. A large proportion of the neutral dysprosium lines appear at about 2200° and are often very faint in the arc. Their absence at the lowest temperature used places them as a rule in class III or III A. Many other lines belonging in class IV A show distinctly in the high-temperature furnace spectrum and must be measured when a complete listing of neutral lines is undertaken. The region under examination contains also a very distinctive group of low-temperature lines, of which the first is \(\lambda\) 4045.977, and the others are near λ 4200. These are distinct and sharp at 2000°, reverse widely in the high-temperature furnace, and are strong in the arc. They are typical class II lines, none of the dysprosium lines in this region being assigned to class I. As Eder has noted, these class II lines are the only ones which show marked widening. Most of the dysprosium lines remain sharp for different excitations, and no evidence of hyperfine structure has appeared.

The ionized lines of dysprosium are easily excited, some appearing in the furnace at temperatures as low as  $2200^{\circ}$ . An arc spectrum of moderate exposure is made up of the ionized and a few of the stronger neutral lines, the furnace or a very strong arc exposure being required to bring up the lines forming the greater part of the neutral spectrum. The arc intensity of these is often so low as to be given as  $1 - 10^{\circ}$  in the table.

Holmium.—The holmium spectrum in this region presents many features of interest. The study is difficult on account of the impure material which the writer and apparently all previous workers have had to use and the unsatisfactory state of the wave-lengths for this

<sup>1</sup> Op. cit., 127, 1, 1918. 2 Loc. cit.

<sup>&</sup>lt;sup>3</sup> Publikationen des Astrophysikalischen Observatoriums zu Potsdam, 20, 60, 1909.

spectrum. Table V contains 164 lines, whose identification with holmium was in nearly all cases clear. The wave-lengths are for the most part those of Exner and Haschek, converted to the international system. Dysprosium lines, which occur very generally in Exner and Haschek's holmium list, were eliminated by means of my material for dysprosium.

Hyperfine structure with wide patterns is very common for both neutral and enhanced lines of holmium, 58 of the lines having either four or six components. Lines given as having four components may be really six-component lines with close patterns. The lines of another large group are distinctly double. A few of the six-component lines were partially resolved, the structure being similar to that of praseodymium<sup>2</sup> in that the spacing of components is graduated, with the larger interval in some cases to the violet, in others to the red. Wide patterns of holmium lines are not confined to lines of high excitation as are those of praseodymium. While the stronger ionized lines in this region are always highly complex, and the strongest neutral lines, a group between \$\lambda 4050 and \$\lambda 4200\$, are single or at most double, a number of low-temperature neutral lines having wide patterns are found. Of these the class I line  $\lambda$  4254.43 is strong in the arc and at all furnace temperatures, but others, of which λ 4497.7 and  $\lambda$  4262.52 are typical, are faint in the arc, but strong at high and medium furnace temperatures.

The neutral spectrum contains the prominent lines of classes I and II, photographed at about 2000°, and, in addition, a large number of class III lines which are often relatively faint in the arc. The latter lines are usually at least double and in the blue region show many good examples of hyperfine structure.

The ionized lines, usually complex, become less frequent near  $\lambda$  4000, some of them having considerable intensity in the high-temperature furnace, as well as in the arc. Their ionized character was readily shown, however, by their disappearance w'n a mixture with caesium was used in the furnace.

CARNEGIE INSTITUTION OF WASHINGTON MOUNT WILSON OBSERVATORY August 1930

<sup>&</sup>lt;sup>1</sup> Some of these having the best resolution appear as if they may have eight components.

<sup>2</sup> Loc. cit.

# MULTIPLET INTENSITIES IN THE SOLAR SPECTRUM<sup>2</sup>

### By R. VAN DER R. WOOLLEY

#### ABSTRACT

The widths of the lines in a titanium multiplet in the solar spectrum have been measured and compared with the theoretical intensities given by the quantum theory, which are in agreement with the observations of laboratory emission spectra. There is a marked divergence in the solar spectrum, the weaker lines appearing relatively stronger. Further, a survey of all the lines in the solar spectrum in the region  $\lambda\lambda$  4500–4600 was made. When the lines are grouped together according to Rowland intensity, the square of the mean width, which gives the number of atoms concerned in the formation of the line, shows a similar marked divergence from the values found by Adams and Russell, who calibrated the Rowland intensities on the basis of theoretical multiplet intensities. Multiplet intensities in the solar spectrum differ widely from intensities in emission spectra, the weaker lines being relatively stronger in the solar spectrum.

Using the 150-foot solar tower and 75-foot spectrograph, the writer has made a photometric survey of the lines in a small region in the solar spectrum from  $\lambda$  4500 to  $\lambda$  4600. The work is preliminary and supplements the projected Mount Wilson survey of the solar spectrum. The method of standardizing the photographic plates is different from that used by St. John in his investigation of selected multiplets, and utilizes apparatus designed by Dunham, which is equivalent to a step wedge placed in front of a spectrograph. The procedure adopted was to cut a plate in half. One half was exposed to the solar spectrum and the other in the standardizing spectrograph, both halves being developed together. The contours of the absorption lines were then measured with a microphotometer.

All but the strongest lines are, however, very narrow. Hence, on account of the finite resolving power of the original spectrograph and also of the microphotometer, the contour so determined is necessarily only approximate. The limitations of the method and various corrections have frequently been discussed elsewhere, and some experimenters are devoting themselves to increasing the accuracy of the method; but a great deal of information, particularly about the stronger lines, can be obtained with the methods at present

<sup>&</sup>lt;sup>2</sup> Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington, No. 413.

available. In particular, the widths of the lines, in thousandths of an angstrom, taken at some definite point (e.g., where the intensity has fallen to nine-tenths that of the continuous background), can be determined for the stronger lines. These widths have been used by A. Unsöld to determine the "number of atoms above I sq.cm of the photosphere." Milne has attached a definite meaning to this phrase, I namely, the number of atoms above a certain optical depth which depends on the intensity, expressed as a fraction of the intensity of the continuous background, at which the width is measured.

The immediate application of the present set of observations is to examine the calibration of Rowland's scale of intensities by Russell and Adams, and to test the underlying assumption that the intensities of lines in a multiplet are proportional to the corresponding intensities in laboratory emission spectra. The present observations give the important result that there is considerable divergence, the weaker lines being relatively much stronger in the solar spectrum than in emission spectra. This result has been suspected by St. John and others for some time.

Individual observations of the width of a line are subject to considerable uncertainty. The chief sources of error and methods of combating them are set out later, but whether a homogeneous set of observations contains systematic errors or not, the relative widths of the lines have a definite meaning, namely, that the squares of the relative widths at some definite point are proportional to the number of atoms above a certain optical depth, systematic errors introducing an uncertainty into the precise value of this depth. Hence the relative intensities in a multiplet may be taken from a homogeneous set of observations (i.e., observations derived from the same plate) with considerably more confidence than can be attached to any individual observation. The same remark holds for statistical observations on the Rowland intensities.

This expected result is borne out by some sets of observations given in Table II. Details of the standardizing apparatus are indicated by Figures 1 and 2. The apparatus was constructed by Dr. Dunham and used by him in his stellar work. The design fol-

<sup>1</sup> Monthly Notices of the Royal Astronomical Society, 89, 3, 1928.

lows the work of G. Hansen<sup>t</sup> and R. Frerichs.<sup>t</sup> Light from an iron arc A falls on a system B consisting of two sheets of ground glass and a condensing lens, after which it falls on a "raster" C. The raster is a brass plate from which a number of horizontal strips



have been cut. A different fraction of the space so formed is blocked in again with black paper in each strip, so that each presents a different open area and passes a different amount of light from the arc. Owing to the ground glass, the illumination of the raster is

> uniform, and the amount of light passed by each strip is proportional to the open area.

> A cylindric lens D focuses the strips vertically but not horizontally on the slit S of a spectrograph, the images being uniform horizontal strips whose intensities are in known ratio. The slit is left open about a millimeter. The images of the lines are therefore wide, and the spectrograph records for each a definite pattern broken up into a number of patches corresponding to the strips on the raster.

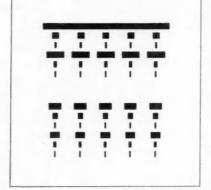


Fig. 2.—The raster C in Fig. 1. The relative areas of the open spaces are, from top to bottom: 100, 25, 5.6; 70, 11.6, 3.1; 49.4, 18.5, 4.4; 35.7, 8.7, 2.0.

The standardizing curve can be made in any wave-length in the iron arc by measuring the photographic densities of these patches. A very wide range of photographic densities may be used by a judicious choice of arc lines different in intensity but close together in wave-length.

A typical curve is shown in Figure 3.

<sup>1</sup> Zeitschrift für Physik, 29, 356, 1924.

<sup>2</sup> Ibid., 31, 305, 1925.

Using Eastman 33 plates, developed four minutes at  $65^{\circ}$  F in Kodak formula D61a developer (half-strength), I have found no distinct variation in the curves for the same plate over a range of wave-lengths from  $\lambda$  4400 to  $\lambda$  4700; probably this is true from  $\lambda$  4200 to  $\lambda$  4900. Provisionally it seems safe to suppose that two plates with the same emulsion number exposed and developed under similar circumstances have the same curve; but this has not been tested thoroughly, and it is better in any case to determine the curve again for each plate used.

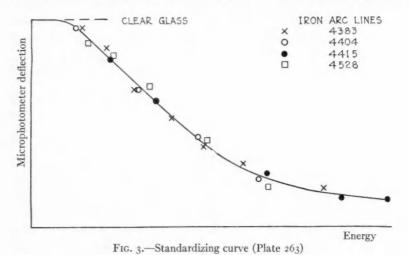
The errors to be expected in this type of observation fall into two classes, photographic and instrumental. Photographic errors are due to (1) errors in standardizing, (2) the presence of chemical fog. and (3) the Eberhard effect. With regard to (1), there should be no difficulty in obtaining a reliable curve if such precautions are observed as making the time of the standardizing exposure equal to the exposure time on the solar spectrum. With regard to (2), it is necessary to have a very clean plate. Washing must be very thorough and the time of development chosen carefully. A fogged plate is useless. With regard to (3), with the very large dispersion used, there should not be much trouble, especially as the observations have been made in different orders and the plates developed with different developers. I have, however, developed some plates with ferrous oxalate, which gives no Eberhard effect, but with which it is difficult to get clear plates. Good results have been obtained by washing very thoroughly after developing and then fixing in an acid hardening bath (Kodak formula F 1).

The instrumental errors are due to finite theoretical resolving power, imperfect focus, the presence of scattered light, ghosts, and lens reflections.

Two gratings have been tested in the present series of observations, one ruled by Michelson and one ruled very recently by Jacomini at the Mount Wilson Observatory. The latter is a much faster grating but does not give as good resolution on absorption lines, although the theoretical resolving power of the two gratings is about the same. The Michelson grating has therefore been used for the observations, but the comparison reminds us to what an extent we are relying on the performance of the grating.

#### OBSERVATIONAL ERRORS

The general method of checking the observational errors has been to examine the line  $\lambda$  4554 of  $Ba^+$  in detail, with the conditions varied as much as possible. The observations on this line are given in Table I. The width  $\lambda - \lambda_0$  of the line is given in thousandths of an angstrom at three points where r, the ratio of the intensity inside the line to that of the continuous background just outside the line, has the values  $\frac{9}{10}$ ,  $\frac{3}{4}$ , and  $\frac{6}{10}$ . The central intensity is also given, as a percentage of the intensity of the continuous background.



With the scale used in making the microphotometer tracings, I mm corresponds to about 0.01 A in width, and the slope of  $\lambda$  4554 is such that an error of I mm in fixing the position of the ordinate  $r = \frac{9}{10}$  introduces an error of about 2 mm (20 units in Table I) in the width of the line. Owing to various irregularities, it is sometimes difficult to fix the position of the continuous background within half a millimeter, and, owing to plate grain, the curve traced by the photometer may have an effective width of a millimeter.

The series of readings in Table I fixes the limitation of the method at about o.o. A for any individual observation.

To test how much of the error is systematic and how much accidental, the three weaker lines given in Table II were observed re-

peatedly. These are good, clean lines, free from blends. The measured widths are given for each spectrogram. The values in parentheses are corrected widths corresponding to a width of 100 for

TABLE I (Unit for  $\lambda - \lambda_0 = 0.001 \text{ A}$ )

PLATE GRATING	Coverno	0		$\lambda - \lambda_0$	CENTRAL INTENSITY	Developer	
	Order	$r = \frac{9}{10}$	34	6 10	(Per Cent)		
W 194	Michelson	2	150	85	70	30	D61a
301	Michelson	3	A 5 154 B 154	97	70 78	3I 29	X-ray X-ray
317	Michelson	3	165	104		26	Oxalate
313	Michelson	3 2	162	108	77 83	33	Oxalate
332	Michelson	2	160	101	82	26	Borax D76
324	Michelson	3	165	116	80	20	Borax D76
350	Michelson	2	160	108	83	25	Oxalate
264	Mt. Wilson	2	152	83	50	50	D61a

#### NOTES TO TABLE I

W 301.—The two spectra A and B were taken on the same plate. The slit on the 75-ft. spectrograph is several inches long, and only a small section is used to make the actual spectrum, which is admitted through a screen near the plate. In B only the section of the slit actually used was exposed to sunlight; in A all was uncovered, thus increasing about threefold the amount of light scattered by the grating. As indicated above, the effect on the observations is negligible.

W 350.—Standardized by a different method (rotating sector).
W 264.—Results obtained with the Mount Wilson grating are given for comparison.

TABLE II

Line	4533 · 25	4534.78	4545.96
Element	Ti	Ti	Cr
Rowland intensity	4	4	3
		$r = \frac{9}{10}$	
W 194	96 (64)	87 (58)	80 (53)
	101 (65)	95 (62)	77 (51)
	104 (64)	100 (62)	90 (55)
	101 (63)	97 (61)	90 (56)
		$r = \frac{3}{4}$	
W 194	61 (72)	50 (59)	47 (55)
	65 (64)	66 (65)	52 (51)
	64 (59)	71 (66)	60 (55)
	61 (56)	61 (56)	61 (56)

 $\lambda$  4554 taken from the same plates. A comparison of the measured and corrected widths shows that the errors are about one-half systematic and one-half accidental. We therefore eliminate part, at least, of the systematic errors when we consider relative widths.

#### OBSERVATIONS ON A MULTIPLET

The best multiplet occurring in the spectral region examined in the present series of measures is titanium  $a^5F - y^5F^\circ$ . In this multiplet no less than eight lines occur free from serious blending in the solar spectrum. For photometry a line must be free from other lines, and it is very difficult to find in the solar spectrum multiplets giving many lines of sufficient intensity (i.e., having a Rowland

TABLE III

	ROWLAND	OBSERVED VALUES OF LOG N*					
LINE	Intensity	W 194	W 301	W 313	W 350	Mean	LOG N
4512.74	3	-0.25		-0.23		-0.24	-0.95
4518.03	3	.21		.18		. 20	0.80
4527.32	3	.25		.13	-0.10	.16	1.00
1533.25	4	.00	0.00	.00	.00	.00	0.00
1534.78	4	.10	06	.03	.04	.06	0.19
1535.58	3	. 28	.16	.23	.18	.21	0.40
1536.06†	2	.21	-34	.08	.08	.18	0.70
4544.70 †	3	-0.10	-0.17	-0.07	-0.10	-0.11	-1.00

<sup>\*</sup> Where no value is recorded, the line does not occur on the plate concerned.

number greater than or equal to 1) and sufficiently clear of other lines.

The relative widths in a homogeneous set of readings were taken at  $r = \frac{9}{10}$ . The square of these widths is proportional to N, the number of atoms above a certain optical depth. The observed values of  $\log N$  thus found are given in Table III. They may be compared with theoretical values given by the quantum theory, which are in general agreement with observations of laboratory emission spectra. The divergence between the observed and theoretical values is very marked, the weaker lines appearing relatively much stronger in the solar spectrum.

<sup>† 4536</sup> and 4544 are slightly blended.

<sup>&</sup>lt;sup>1</sup> Russell, Mt. Wilson Contr., No. 345; Astrophysical Journal, 66, 347, 1927.

<sup>&</sup>lt;sup>2</sup> Russell, Proceedings of the National Academy of Sciences, 11, 314, 322, 1925.

TABLE IV

Your	ROWLAND	ROWLAND ELEMENT -		$\lambda - \lambda_0$			
Line	Intensity	ELEMENT	$r = \frac{9}{10}$	$r=\frac{3}{4}$	$r = \frac{6}{10}$		
1501.28	5	Ti+	150	70	59		
502.23	2	Mn	75	30	1		
508.29	4	Fe	90	50	28		
511.90	I	Cr	50	t			
512.75	3	Ti	72	37	17		
515.34	3	Fe <sup>+</sup>	90	45	25		
		Fe	85				
517.54	3	Ti		38	15		
518.03	3		75	40	25		
520.23	3	Fe <sup>+</sup>	70	40	26		
522.81	2	Ti	70	35	24		
523.41	I	Fe	40				
525.15	5	Fe	120	60	40		
526.93	3	Ca	83	48	26		
527.33	3	Ti	72	42	30		
531.16	5	Fe	104	70	48		
531.63	2	Fe	75	30	4-		
533.25	4	Ti	96	61	37		
533.97	6	Ti+ Co*	120	71			
000 / 1	-	Fe <sup>+</sup>			55		
534.17	1		70	26			
534-79	4	Ti	87	50	35		
535.58	3	Ti	70	45	35		
536.06	2	Ti	75	42	30		
540.51	2	Cr	75	38	t		
540.71	2	Cr	72	35	t		
541.52	2	Cr Fe+	80	35	10		
544.02	I	$Ti^+$	55	20			
544.70	3	Ti	85	40	30 .		
545.15	1	$Ti^+$	60	20			
545.98	3	Cr	80	47	28		
548.77	2	Ti	60	40	30		
	2	Fe			-		
550.78	8	Ba+	70	35	15		
554.04		40.00	150	85	70		
554.46	1	Fe	50				
554-99	2	Cr+	70	10			
555-49	3	Ti	63	35	14		
555.89	3	$Fe^+$	75	40	30		
556.14	4	Fe	105	60	36		
558.65	3	Cr+	90	45	21		
560.10	2	Fe	50	20			
563.77	4	$Ti^+$	150	70	45		
566.53	ī	Fe	50	1			
66.87	ī	Fe	48				
68.77	ī	Fe	70?	t			
71.10	5	Mg	87	60	40		
		Ti <sup>+</sup>	120				
571.98	5			75	50		
574.22	I	Fe	43				
574-73	2	Fe	60	30	t		
576.34	2	Fe <sup>+</sup>	75	35			
78.56	3	Ca	85	45	22		
80.06	3	Cr	80	49	25		

<sup>\*</sup> Bold-face type indicates the dominating element in a blend.

TABLE IV-Continued

Line	ROWLAND	ELEMENT	λ-λ <sub>0</sub>			
LINE	Intensity	ELEMENT	$r = \frac{9}{10}$	$r=\frac{3}{4}$	$r = \frac{6}{10}$	
4580.42	I	V	70	t		
1582.83	1	Fe <sup>+</sup>	60	26		
1583.84	4	$Fe^+$	140?	70	35	
1584.73	I	Fe	35?			
1584.83	2	Fe	35	28		
1585.88	4	Ca	105?	70	47	
1586.38	I	V	50	t		
587.14	2	Fe	60	20		
588.21	3	Cr+	85	35	14	
589.95	3	Ti+	95	50	23	
591.40	2	Cr	65	35	12	
592.06	1	Cr+	60	t		
592.53	2	Ni	80	40	10	
1592.66	4	Fe	90	50	40	
594.13	2	NV	60	t		
595-37	2	Fe	70	33	t	
596.07	2	Fe	70	30		
596.42	1	Cr+	42			
598.13	3	Fe	77	45	20	
598.84	2	Fe?	57	28		
599-37	2	N	50	25		
599.76	3	Cr	70	40	26	
602.01	3	Fe	60	35	10	
602.95	6	Fe	90	63	36	

# observations in the interval $\lambda\lambda$ 4500–4600

These observations, given in Table IV, are a homogeneous set, that is to say, they are taken from the same plate (W 194) and reduced with the same standardization curve. The wave-length, Rowland intensity, and element are given, followed by the width  $\lambda - \lambda_0$  at  $r = \frac{9}{10}$ ,  $\frac{3}{4}$ , and  $\frac{6}{10}$ . All the identified lines of Rowland intensity 1 or greater in the range  $\lambda\lambda$  4500–4600 are given, except those too badly blended to be measurable.

# THE ADAMS-RUSSELL CALIBRATION OF ROWLAND'S SCALE

In the region observed,  $\lambda\lambda$  4500–4600, there are 17 lines of Rowland intensity 1, 21 lines of intensity 2, 20 lines of intensity 3, and 8 of intensity 4. Taking the measures of plate W 194 at  $r = \frac{9}{10}$ , we find the arithmetic mean widths of the lines grouped according to their Rowland numbers to be as shown in the second column of Table V. The square of the relative width gives the number N. The resulting observed values of log N in the second column of

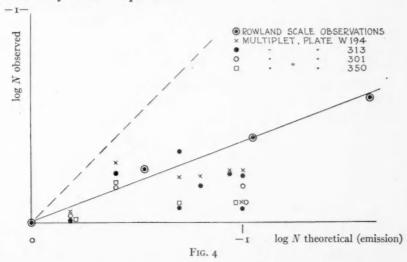
Table V are to be compared with the values given by Adams and Russell, which appear in the last column.

The divergence between the two series is, of course, an illustration of the general phenomenon of the difference in relative multiplet

TABLE V

Down		$\log N$			
ROWLAND INTENSITY	MEAN WIDTH	Observed	Adams and Russell		
I	0.053	0.00	0.00		
2	.067	. 20	0.54		
3	.078	-34	1.06		
4	0.106	0.60	1.59		

intensities between absorption and emission spectra, since Adams and Russell derived their values of N by using the assumption that the absorption lines have the same relative strengths as the lines in laboratory emission spectra.



This result seems to be established by the present series of observations beyond the limits of possible observational error. The values are shown graphically in Figure 4, the values of  $-\log N$  as ob-

<sup>&</sup>lt;sup>1</sup> Mt. Wilson Contr., No. 358; Astrophysical Journal, **68**, 1, 1928. To be distinguished carefully from their analysis of stellar spectra (Mt. Wilson Contr., No. 359; Astrophysical Journal, **68**, 9, 1928).

served in this series being plotted vertically against the theoretical (or emission) values of  $-\log N$  which are entered horizontally. The points all fall unmistakably below the broken line representing equality between the two values.

The observed values of  $-\log N$  for the Rowland intensities are plotted vertically (the o being shifted to give  $\log N = 0$  for R = 4) against the values of Adams and Russell. The slope of the line joining them is roughly the same as that for the individual multiplet, as one would expect. The weaker multiplet lines show some divergence from the Adams-Russell line, which may be a real characteristic of the multiplet or may be due to the greater difficulty of measuring weaker lines.

It should be noted that the above divergence between theory and observation assumes the correctness of Unsöld's formula for the atomic scattering coefficient and the Unsöld-Milne theory of the formation of absorption lines. The present observational results may, alternatively, be regarded as a failure of Unsöld's theory. On this view, the multiplet intensities are normal and the Adams-Russell calibration is correct; but Unsöld's method deduces the wrong number of atoms from the relative widths of the lines.

It is a pleasure to record my gratitude to the Director of the Mount Wilson Observatory, to the staff generally, and to Dr. Dunham especially, for apparatus, help, and advice. My best thanks are due to Miss Ware for assistance in preparing the photometric curves. I am indebted to the Commonwealth Fund for the Fellowship which has made it possible for me to work at Mount Wilson.

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# INTENSITIES IN STELLAR SPECTRA OF A TRIPLET OF Si III

BY OTTO STRUVE AND C. T. ELVEY

#### ABSTRACT

It is shown that under certain assumptions the ratios of *total absorptions* of the members of the Si III triplet  $(2^1s-3^3p)$ ,  $\lambda\lambda$  4553, 4568, and 4575, in stellar spectra, depend upon the *form of the absorption coefficient*. If the latter is caused by radiation damping, then the total absorptions are proportional to the square roots of the laboratory intensities. If thermal agitation or any other cause produces a flat but extended absorption coefficient, then the total absorptions are directly proportional to the laboratory intensities.

Observations of seven stars confirm the square-root relationship. This is in agreement with the assumption that the absorption coefficient is due to radiation damping, and tends to confirm the hypothesis that dish-shaped lines in early type stars are due to rotation and not to an unusual form of the absorption coefficient.

The intensities in emission were observed in the spark between terminals of fused silicon. The observed ratios are 4.0:2.4:1, while the theoretical rules give 5:3:1. The deviation may be real.

It is noted that stellar spectra are relatively more suitable for the study of faint members of multiplets than are laboratory sources.

The stellar intensity ratios are independent of the absolute magnitude of the stars.

#### I. INTRODUCTION

Let us consider the following problem: Given the ratios of the total intensities of two or more lines in emission, what are the corresponding ratios of the total absorptions of the same lines when observed in stellar spectra?

Let  $N_i$  be the number of atoms, in a gas, per cubic centimeter in state i, and let  $A_{ij}$  be the probability that an atom in state i will, in unit time, fall to a lower energy-level j, with emission of energy. Then the total intensity of the emission line is

$$I = h\nu \cdot N_i A_{ij} \cdot V , \qquad (1)$$

where V is the volume of the gas and NA is the number of atoms per cubic centimeter that actually make the transition  $i \rightarrow j$ . The intensity of the line is proportional to the number of acting atoms. Consequently, if several lines have been observed,

$$I_1:I_2:I_3...=N_1A_1:N_2A_2:N_3A_3...$$

<sup>1</sup> Since, according to Einstein, emission takes place spontaneously, as well as induced by radiation,  $A_{ij} = a_{ij} + b_{ij} \cdot u$ , where a and b are Einstein's coefficients for emission and u is the density of radiation.

provided that approximately  $\nu_1 = \nu_2 = \nu_3 = \dots$ . Formula (1) holds as long as the emitting layer is too thin to give an appreciable amount of self-reversal.

If we observe a triplet originating in transitions from three upper levels  ${}^{3}p_{1}$ ,  ${}^{3}p_{2}$ , and  ${}^{3}p_{3}$  to a single  ${}^{1}s$ -level, then in equilibrium conditions,

$$N_{x} = cg_{1}e^{-\frac{E_{x}}{KT}},$$
  
 $N_{z} = cg_{2}e^{-\frac{E_{z}}{KT}},$   
 $N_{3} = cg_{3}e^{-\frac{E_{3}}{KT}},$ 

where c is a constant, g is the statistical weight of the state, and T is the temperature. The energies of the three states are approximately the same:  $E_1 \approx E_2 \approx E_3$ . Consequently,

$$I_1:I_2:I_3=g_1A_1:g_2A_2:g_3A_3$$
.

According to L. S. Ornstein and H. C. Burger,  $^{\text{\tiny T}}A_1=A_2=A_3$ . Therefore

$$I_1:I_2:I_3=g_1:g_2:g_3$$
.

The case of an absorption line is more complicated. The absorption coefficient,  $\sigma$ , is in general a function of the difference in wavelength  $(\lambda - \lambda_0)$  between the center of the line and the point considered. It is also proportional to the number of atoms that pass from state j to state i, or  $N_j B_{ji}$ , where  $N_j$  is the number of atoms per cubic centimeter in state j and  $B_{ji}$  is the corresponding probability for absorption

$$\sigma = C\varphi(\lambda - \lambda_0) N_i B_{ii} , \qquad (2)$$

where C is a constant depending upon the central wave-length,  $\lambda_0$ , and upon various properties of the atom and electron. The residual intensity at any given point  $(\lambda - \lambda_0)$  is itself a function of  $\sigma$ :

$$I_{\lambda} = I_{0} \psi(\sigma)$$
, (3)

<sup>1</sup> Zeitschrift für Physik, 24, 41, 1924.

where  $I_0$  is the intensity outside the line. The form of  $\psi$  is not very essential. In the case of true absorption, where the energy collected by the atoms is not re-emitted in the same wave-length,

$$I_{\lambda} = I_{0}e^{-\int \sigma dl}$$
 or  $I_{\lambda} = I_{0}e^{-\sigma l}$ , (4)

if  $\sigma$  is independent of the thickness of the layer l. In a scattering atmosphere

$$I_{\lambda} = I_{0} \frac{1}{1 + \sigma l}, \qquad (5)$$

according to the Schuster-Schwarzschild approximation. A. Unsöld has shown that the difference between the two forms of  $\psi$  is not great and we shall here adopt formula (4), although all the computations could also have been made with respect to formula (5). In this connection it is well to remember that in (4) we altogether ignore re-emission from i to j, while in (5) we take into consideration only that part of the re-emission that results from atoms that have reached state i by the transition  $j \rightarrow i$ . Eddington has shown that this is not necessarily correct. If state i can be reached by other transitions and if there is an excess of re-emission  $i \rightarrow j$ , then the intensities can be shown to be altogether different from those which result under the assumption that only transitions of the type  $j \gtrsim i$  are possible. With this reservation in mind we consider the value of  $\sigma$ . Unsöld has shown that in the case of pure radiation damping

$$\sigma = \frac{2\pi e^4 \lambda_0^2 \Re}{3m^2 c^4 (\lambda - \lambda_0)^2}.$$
 (6)

Here  $\mathfrak R$  designates the number of classical oscillators per cubic centimeter. But according to R. Ladenburg,<sup>3</sup>

$$\Re = N_i f = N_i B_{ii} h \nu \frac{m}{\pi e^2},$$

where f is called the "oscillatory strength."

<sup>1</sup> Ibid., 59, 363, 1930.

<sup>&</sup>lt;sup>2</sup> Ibid., 44, 793, 1927; 46, 768, 1928.

<sup>3</sup> Ibid., 4, 454, 1921.

Substituting (6) in (4), we get for the total energy absorbed in the line

$$E = I_0 \int_{-\infty}^{+\infty} [\mathbf{1} - \psi(\sigma)] d\lambda = 2\sqrt{\pi} \sqrt{CN_i B_{ii}},$$

where C is a new constant. The ratios of the total absorptions are proportional to  $VN_jB_{ji}$ . Consequently, we find that while in emission the intensities are proportional to  $N_i$ , in absorption they are proportional to  $VN_jB_{ji}$ . The numbers  $N_i$  and  $N_j$  may or may not be in a definite ratio to each other. Consider again the case of a triplet having a single lower level,  $S_i$ , and three close upper levels,  $S_i$ ,  $S_i$ 

$$E_1: E_2: E_3 = \sqrt{I_1}: \sqrt{I_2}: \sqrt{I_3}$$
 (7)

Radiation damping is not in all cases effective in defining  $\sigma$ . Various causes are known that produce flat and extended absorption coefficients, resulting, of course, in wide and shallow lines. Thermal agitation or Stark effect may serve as examples. In the stars we frequently observe "dish-shaped" contours, and it is of interest to ascertain whether or not such lines are produced by a failure of (6) in the expression for the absorption coefficient. It may be noted that dish-shaped contours have been attributed by us in former papers² to the effect of rotation in the stars, and the outcome of the present investigation is in full accord with this interpretation.

Suppose that  $\varphi$  in (2) is so small, even in the center of the line, that  $\sigma l$  is a small quantity. Then in (4),  $e^{-\sigma l}$  may be replaced<sup>3</sup> by  $1-\sigma l$  and

$$E = I_0 \int_{-\infty}^{+\infty} \sigma l \, d\lambda.$$

<sup>&</sup>lt;sup>1</sup> We are making here the obvious assumption that the thickness of the absorbing layer, *l*, is the same for all members of our multiplet. That this is justified, results from the investigations of C. E. St. John on multiplets in the solar spectrum (*Astrophysical Journal*, 70, 312, 319, 1929).

<sup>&</sup>lt;sup>2</sup> Elvey, Astrophysical Journal, 71, 221, 1930; Struve, ibid., 72, 1, 1930.

<sup>&</sup>lt;sup>3</sup> Formula (8) was first derived by Unsöld, and was used by him in a joint paper by Unsöld, Struve, and Elvey on the interpretation of the interstellar Ca II lines, soon to appear in Zeitschrift für Astrophysik.

Butz

 $\int \sigma d\lambda = N_i B_{ii} \times \text{Const.}$ 

Consequently,

 $E \sim N_i B_{ii}$ .

Therefore in the case of a flat and broad curve for  $\varphi$  ( $\lambda - \lambda_0$ ) we should observe in our triplet ( ${}^{r}s - {}^{3}p$ ) the following ratios:

$$E_1: E_2: E_3 = I_1: I_2: I_3$$
 (8)

#### II. OBSERVATIONS

The well-known triplet of Si III at  $\lambda\lambda$  4553, 4568, and 4575 is particularly suitable for a test of the two relations (7) and (8). According to A. Fowler,<sup>2</sup> these lines arise from transitions between a single  $2^{t}s$ -level and three close  $3^{3}p$ -levels. It is therefore the simplest form of triplet in the triplet system of Si III, and the rules of Burger and Dorgelo give for the theoretical ratios of the intensities in emission<sup>3</sup>

$$I_{4553}:I_{4568}:I_{4575}=5:3:1$$
.

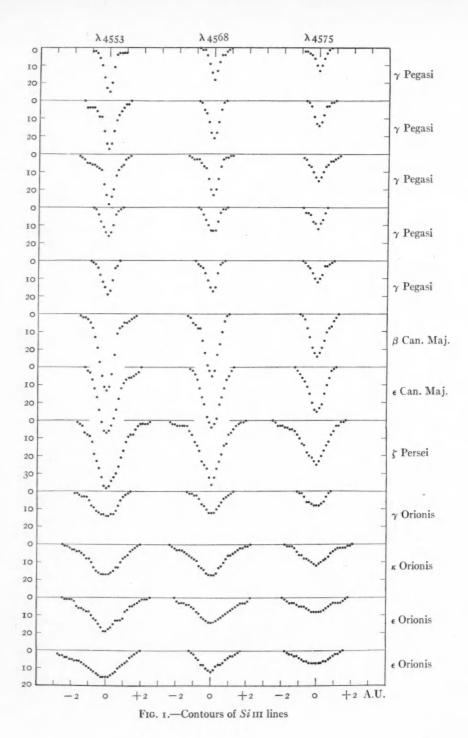
The stellar intensities of the corresponding absorption lines were determined in the usual way with the microphotometer. Fine-grained Eastman Process plates standardized with a tuve sensitometer were used for all stars. The linear dispersion was 10A per millimeter at  $\lambda$  4500. Contours for eleven plates of seven stars are shown in Figure 1. The actual shape of the contour, in stars with narrow lines, is largely due to the instrument, and it would be futile to attempt a direct comparison with Unsöld's theoretical contour. The total absorptions of the lines were found by measuring the areas of the curves with a planimeter. A summary is given in Table I, where the stars are arranged in order of increasing line width. The corresponding components of the equatorial rotational velocities, in the line of sight, are given in the last column. There is no appreciable

<sup>1</sup> R. C. Tolman, Statistical Mechanics, p. 175, 1927.

<sup>&</sup>lt;sup>2</sup> Philosophical Transactions of the Royal Society of London, A, 225, 1, 1924. See also W. Grotrian, Graphische Darstellung der Spektren, 2, 61, 1928.

<sup>3</sup> R. Frerichs, Handbuch der Physik, 21, 443, 1929.

<sup>&</sup>lt;sup>4</sup> These values were taken from C. T. Elvey, op. cit., p. 227, 1930, except for κ Orionis, which was not included in that paper and for which an estimate was made for the purpose of this paper.



systematic run of the values of  $E_1: E_2: E_3$  with width, and the mean agrees reasonably well with  $\sqrt{5}: \sqrt{3}: 1$ .

The intensities of the Si III lines in emission have not, to our knowledge, been measured. Therefore we have attempted to obtain the values of  $I_1:I_2:I_3$  by direct observation. A condensed spark, with self-induction, was passed between terminals of fused silicon.

TABLE I Intensities of Si iii Lines

Star	PLATE	Intensity in Angstrom Units of Complete Absorption			RAT	ROTA- TIONAL VELOCITY		
		λ 4553	λ 4568	λ 4575	λ 4553	λ 4568	λ 4575	VELOCITY
								km/sec.
γ Pegasi	R 1588	0.179	0.114	0.079	2.3	1.4	1.0	0
	1596a	.252	.134	.097	2.6	1.4	I.0	
	1596b	. 280	. 185	.143	2.0	1.3	1.0	
	1599	.116	.098	.081	1.4	1.2	1.0	
	1823	.143	.114	.088	1.6	1.3	1.0	
$\beta$ Can. Maj	1771	.475	.340	.243	2.0	1.4	I.0	12
€ Can. Maj	1626	.492	.380	.261	1.9	1.5	I.0	12
Persei	1660	.655	.520	.405	1.6	1.3	1.0	55
$\gamma$ Orionis	1600	.232	.145	.094	2.5	1.5	I.0	65
κ Orionis	1826	.358	-345	.195	1.8	1.8	1.0	100
€ Orionis	1736	.400	.300	.161	2.5	1.9	1.0	100
	1825	0.384	0.164	0.136	2.8	I.2	1.0	,
Mean Probable er-					2.1	1.4	1.0	
ror of mean Probable er- ror of one					±0.09	±0.04		
plate					±0.30	±0.15		

Plate 1596 was reduced twice from two independent microphotometer tracings. The two sets of values given in the table were treated as separate determinations in taking the mean. Experiments have shown that the width of the analyzing slit of the microphotometer had no effect upon the results.

The lines of Si III are present, but they are appreciably broadened in comparison to the lines of iron which appear as impurity. The central intensities of the lines of Si III were determined with the microphotometer, the results for four plates appearing in Table II. There seems to be a small but real deviation from the theoretical intensities, but the laboratory values remain greatly in excess of those obtained from the stars. In fact, the square roots of the ob-

served laboratory intensities fit the stellar values as well as do the theoretical intensities.

#### III. CONCLUSIONS

The observed stellar intensities are in the following ratio: 2.1: 1.4:1.0. If we compare this with the square roots of the theoretical intensities, 2.24:1.73:1.00, or with the square roots of the observed emission lines in the spark, 2.0:1.5:1.0, we can conclude that the stellar intensities follow approximately the relationship expressed in (7):  $E_1:E_2:E_3=\sqrt{I_1}:\sqrt{I_2}:\sqrt{I_3}$ , which agrees with the assumption that  $\sigma$  is given by radiation damping, as in (6). This appears to be

TABLE II

Intensities of Si iii Lines in Spark

PLATE	INTENSITY II	Magnitudes	es of Stellar	RATIO OF INTENSITY			
	λ 4553	λ 4568	λ 4575	λ 4553	λ 4568	λ 4575	
	1.45	1.08	0.00	3.80	2.70	1.00	
	1.53	1.02	.00	4.05	2.55	I.00	
	1.57	0.90	.00	4.20	2.30	I.00	
	1.47	0.72	0.00	3.85	1.95	1.00	
Mean	1.50	0.93	0.00	3.98	2.38	1.00	

independent of the character of the lines: wide and shallow lines obey the relationship as well as narrow and deep lines. Evidently our dish-shaped contours are not due to a change in  $\sigma$ . This is of considerable importance, as it tends to confirm our former interpretation of these lines as being due to rotation. Indeed, in the case of rotation the total absorption remains unaffected, though the contours are greatly flattened.

The existence of (7) leads to the interesting result that within a multiplet the faint lines are relatively much stronger in the stars than in the laboratory. Thus if in the laboratory  $I_1:I_2=100$ , the ratio of the stellar lines is only  $E_1:E_2=10$ . It is clear that the stars are relatively more suitable for the detection of faint members in various multiplets than are the laboratory sources.<sup>2</sup>

<sup>&</sup>lt;sup>1</sup> Struve, ibid., 72, 13, 1930.

<sup>&</sup>lt;sup>2</sup> For example, in 7  $\epsilon$  Aurigae many of the faintest multiplet lines of Ti II, classified by H. N. Russell (Astrophysical Journal, 66, 283, 1927), are well visible, and a careful

It is probably permissible to extend relationship (7) to any set of lines, irrespective of multiplet character, provided that the conditions of excitation in the laboratory source and in the reversing layer of the star are similar, and provided that the restrictions mentioned in section I are obeyed. If this is the case, then it is clear that the decline in intensity within any given spectral series, such as the Balmer lines of hydrogen, must be much slower in the stars than in the laboratory. This may in part explain the ease with which the higher members of spectral series are observed in the stars.

Formula (7) may help also to clear up a difficulty in the interpretation of Stark effect in B-type stars. It was found by Struve<sup>2</sup> that the ratio of the total absorption of the forbidden He line  $\lambda$  4470 to that of the permitted line at  $\lambda$  4472, in  $\gamma$  Pegasi, is  $E_{(4470)}$ :  $E_{(4472)} = 0.2$ . In the laboratory<sup>3</sup>  $I_{(4470)}$ :  $I_{(4472)} = 6.6 \times 10^{-10} \cdot F^2$ , where F, the average field strength, is expressed in volts per centimeter. If the foregoing ratio is directly substituted in this equation, the average field is F =17,000 volts per centimeter, a value considerably in excess of the maximum value found by other methods (103 volts/cm  $< F < 10^4$ volts/cm).4 If we use (7) we get for  $I_{(4470)}/I_{(4472)} = 0.04$ , and the corresponding value of F is 8000 volts/cm. It should be noted, however, that the helium lines are somewhat broadened by Stark effect, and the absorption coefficient is not strictly that given by (6). We might therefore expect that for these lines a law intermediate between (7) and (8) should apply, leading to a value of F intermediate between 8000 and 17,000 volts/cm.

Our list of stars, Table I, contains giants of great luminosity,

study of the fainter lines of this star promises interesting results (see also C. H. Payne, *Harvard College Observatory Bull.* 855, 1928). Several other cases are known where predicted multiplet lines of various elements were observed in stellar or solar spectra (Th. Dunham, Jr., and C. E. Moore, *Astrophysical Journal*, 68, 37, 1928; C. E. Moore, and H. N. Russell, *ibid.*, 151, 1928).

<sup>&</sup>lt;sup>I</sup> It may be noted that while the Balmer absorption lines in stars usually extend far out into the violet, the Balmer emission lines in Be stars rapidly decline in intensity from Ha toward higher members, and are rarely seen beyond He.

<sup>&</sup>lt;sup>2</sup> Astrophysical Journal, 70, 237, 1929.

<sup>&</sup>lt;sup>3</sup> J. M. Dewey, *Physical Review*, **28**, 1108, 1926; **30**, 770, 1927. See also correction in *Astrophysical Journal*, **70**: 239, 1929.

<sup>4</sup> Astrophysical Journal, 69, 173, 1929.

such as  $\zeta$  Persei, and dwarfs of small luminosity, such as  $\gamma$  Pegasi.<sup>1</sup> The ratio of intensities seems to be independent of luminosity (or absolute magnitude).

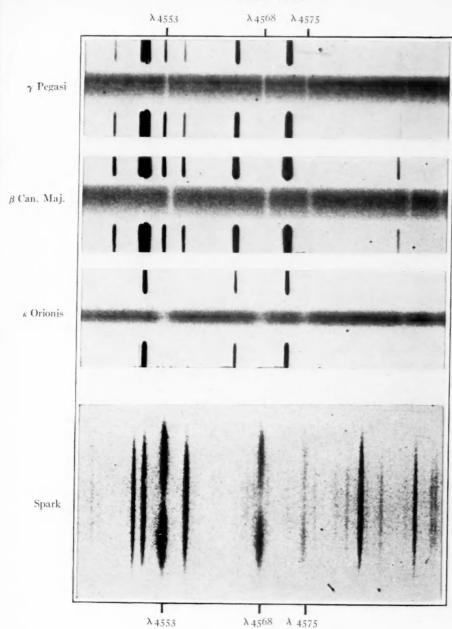
The values from individual plates in Table I show considerable scattering, even for the same star ( $\gamma$  Pegasi). It is very improbable that this is due to real changes in the star, and we are inclined to interpret it as being due to errors of measurement. Whether or not individual stars deviate from the mean by small amounts cannot be decided from the material at hand.

Several stellar spectra and one laboratory spectrum are reproduced in Plate XIII. The widths of the stellar absorption lines are not the same in the three stars:  $\gamma$  Pegasi has very sharp and narrow lines, devoid of rotational broadening; in  $\kappa$  Orionis, on the other hand, the broadening is very pronounced. Our estimate for the component in the line of sight of the equatorial velocity of rotation in this star is 100 km/sec. In the laboratory spectrum the line joining the terminals of the spark was parallel to the slit and the spark was focused upon the slit jaws. The iron lines appear sharp and are most intense near the middle; the Si III lines, on the other hand, show pressure broadening, and are weak in the middle. It is easily seen that the ratio of intensity of  $\lambda$  4553 to that of  $\lambda$  4575 is much greater in the spark than in the stars.

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<sup>1</sup> This is inferred from (1) the appearance of the helium lines, (2) the intensity of the forbidden line of He,  $\lambda$  4470, and (3) the width of the hydrogen lines (*ibid.*, 70, 89, 1929).





LINES OF Si III IN STARS AND IN SPARK

